Question 6 (50 marks)

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

(a) If the amount of the first annual payment is A, write down, in terms of A, the amount of the second, third, fourth and 26th payments.

1st payment (now): _____A_____

2nd payment: _____

3rd payment: _____

4th payment: _____

⋮   ⋮

26th payment: _____

(b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A.

(c) Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of $21·5 million.

(d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4·78%. We want to find the cash value of the prize referred to in part **(c)**.

(i) Complete the table below to show the actual amount and the present value of each of the first three annual payments.

payment number	time to payment (years)	actual amount	present value
1	0		
2	1		
3	2		

(ii) Write down, in terms of *n*, an expression for the present value of the *n*th annual payment.

(iii) Find the amount of prize money payable under the *cash-value* option. That is, find the total of the present values of the 26 annual payments.

Give your answer in millions, correct to one decimal place.

(e) The jackpot described in parts **(c)** and **(d)** above was won by an Irish woman earlier this year. She chose the *cash-value* option. After tax, she received $7·9 million. What percentage of tax was charged on her winnings?