Answer both Question 5 and Question 6.

## Question 5

Gold jewellery is made from a gold alloy - that is, a mixture of pure gold and other metals. The purity of the material is measured by its "carat rating", given by the formula

$$
c=\frac{24 m_{g}}{m_{t}}
$$

where
$c=$ carat rating
$m_{g}=$ mass of gold in the material
$m_{t}=$ total mass of the material.
A jeweller is recycling old gold jewellery. He has the following old jewellery in stock:
147 grams of 9 -carat gold
85 grams of 18-carat gold.
He can melt down this old jewellery and mix it in various proportions to make new jewellery of different carat values. The value of the old jewellery is equal to the value of its gold content only. Gold is valued at $€ 36$ per gram.
(a) What is the total value of the jeweller's stock of old jewellery?

(b) The jeweller wants to make a 15-carat gold pendant weighing 21 grams. He melts down some 9 -carat gold and some 18 -carat gold to do this. How many grams of each should he use in order to get the 21 grams of 15 -carat gold?


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(c) The other metals in the gold alloy are copper and silver. The colour of the alloy depends on the ratio of copper to silver. In all of the old jewellery, the amount of silver is equal to the amount of copper. The jeweller has a stock of pure silver that he can add to any mixture. He wants to make an item that:

- weighs 48 grams
- is of 15 -carat gold purity
- has twice as much silver as copper.
(i) How many grams of copper will this item contain?

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(ii) How many grams of each type of stock (9-carat gold, 18-carat gold, and pure silver) should the jeweller use in order to make this item?


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(d) A large jewellery business makes and sells 14-carat gold wedding rings, weighing an average of 5 grams each. The cost of producing each ring is $€ 135$ plus the value of the gold. The manager has noted that the more they charge for the rings, the fewer they sell. In particular:

- if they charge $€ 200$, they sell an average of twenty per month
- for each additional $€ 20$ charged, the number sold drops by one per month.
(i) Taking the price charged as $€(200+20 x)$, find an expression in $x$ for the monthly profit from these rings.

(ii) Find the range of selling prices for which the monthly profit is at least $€ 1600$.


