Question 4 **(25 marks)**

In a science experiment, a quantity $Q(t)$ was observed at various points in time t. Time is measured in seconds from the instant of the first observation. The table below gives the results.

t	0	1	2	3	4
$Q(t)$	2·920	2·642	2·391	2·163	1·957

Q follows a rule of the form $Q(t) = Ae^{-bt}$, where A and b are constants.

(a) Use any two of the observations from the table to find the value of A and the value of b, correct to three decimal places.

$$Q(0) = Ae^{0} = 2{\cdot}92$$

$$\therefore A = 2{\cdot}920$$

$$Q(t) = 2{\cdot}92e^{-bt}$$

$$Q(1) = 2{\cdot}92e^{-b} = 2{\cdot}642$$

$$e^{-b} = \frac{2{\cdot}642}{2{\cdot}92}$$

$$-b = \log_e \frac{2{\cdot}642}{2{\cdot}92}$$

$$b = 0{\cdot}100$$

(b) Use a different observation from the table to verify your values for A and b.

$$Q(t) = 2{\cdot}92e^{-0{\cdot}1t}$$
$$Q(2) = 2{\cdot}92e^{-0{\cdot}2} = 2{\cdot}391$$

From the table, $Q(2) = 2{\cdot}391$, thus verifying the values for A and b.

(c) Show that $Q(t)$ is a constant multiple of $Q(t-1)$, for $t \geq 1$.

$$\frac{Q(t)}{Q(t-1)} = \frac{Ae^{-bt}}{Ae^{-b(t-1)}}$$
$$= e^{-b} \quad \text{(a constant)}$$

Or

$$\frac{Q(t-1)}{Q(t)} = e^{b}$$

Or

$$Q(t) = 2{\cdot}92e^{-0{\cdot}1t}$$
$$Q(t-1) = 2{\cdot}92e^{-0{\cdot}1(t-1)}$$
$$\frac{Q(t)}{Q(t-1)} = \frac{2{\cdot}92e^{-0{\cdot}1t}}{2{\cdot}92e^{-0{\cdot}1(t-1)}} = \frac{1}{e^{0{\cdot}1}}$$

(d) Find the value of the constant k for which $Q(t+k) = \frac{1}{2}Q(t)$, for all $t \geq 0$.

Give your answer correct to two decimal places.

$$Q(t+k) = \frac{1}{2}Q(t)$$
$$Ae^{-b(t+k)} = \frac{1}{2}Ae^{-bt}$$
$$2e^{-b(t+k)} = e^{-bt}$$
$$2e^{-bk} = 1$$
$$e^{bk} = 2$$
$$bk = \log_e 2$$
$$k = \frac{1}{b}\log_2 e$$
$$k = 10\log_e 2$$
$$k \approx 6{\cdot}93$$