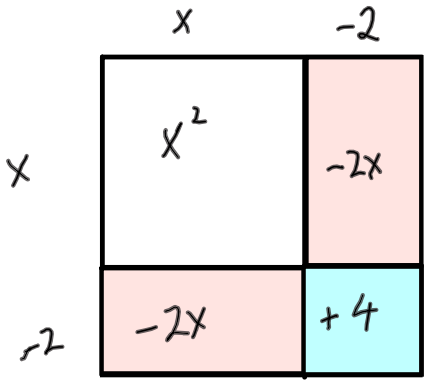
Complete square



$\text{Area} = (x-2)^2$

$\text{Area} = x^2 - 4x + 4$

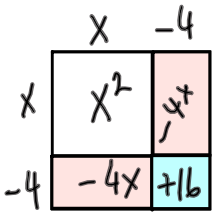
Section 2.6 Max and Min of Quadratic graphs

Example 1

Complete the square on each of the following quadratic expressions. Hence find the minimum value of each expression.

(i) $x^2 - 8x + 10$ (ii) $4x^2 + 4x + 2$

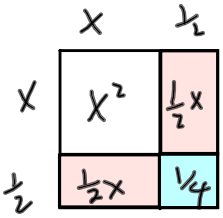
$x^2 - 8x + 10$



$x^2 - 8x + 16 - 16 + 10$

$(x-4)^2 - 6$

$4[x^2 + x + \frac{1}{2}]$



$4[x^2 + x + \frac{1}{2} - \frac{1}{4} + \frac{1}{2}]$

$4[(x+\frac{1}{2})^2 + \frac{1}{4}]$

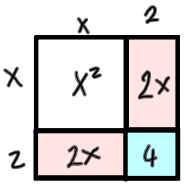
$4(x+\frac{1}{2})^2 + 1$

Aside

Example 2

Write the quadratic equation $x^2 + 4x + 1$ in the form $(x - p)^2 + q$ and hence,
(i) find the minimum point and minimum value of $x^2 + 4x + 1$
(ii) solve the equation $x^2 + 4x + 1 = 0$, leaving your answer in surd form.

(i)



$x^2 + 4x + 1$
$= x^2 + 4x + 4 - 4 + 1$
$= (x+2)^2 - 3$

in form $(x-p)^2 + q = (x + (-2))^2 + (-3)$

min pt $(2, -3)$
min value $= -3$

(ii) Solve?

$(x+2)^2 - 3 = 0$
$(x+2)^2 = 3$
$x + 2 = \pm\sqrt{3}$
$x = -2 \pm \sqrt{3}$

Aside

Complete Square

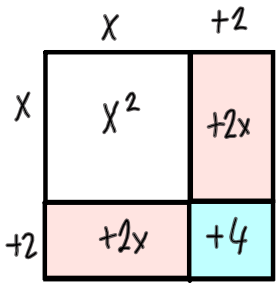
if $y = (x-p)^2 + q$

then the max/min Value $= q$

and this occures when $x = p$

Section 2.6 Max and Min of Quadratic graphs

3. Write each of the following in the form $(x - p)^2 + q = 0$.
(i) $x^2 + 4x - 6 = 0$



$x^2 + 4x - 6 = 0$

$x^2 + 4x + 4 - 4 - 6 = 0$

$(x+2)^2 - 10 = 0$ ✓

$(x - (-2))^2 + (-10) = 0$

min. $(-2, -10)$

THIS CAN ALSO BE FOUND BY DIFFERENTIATION

$y = x^2 + 4x - 6$

$\frac{dy}{dx} = 2x + 4 = 0$

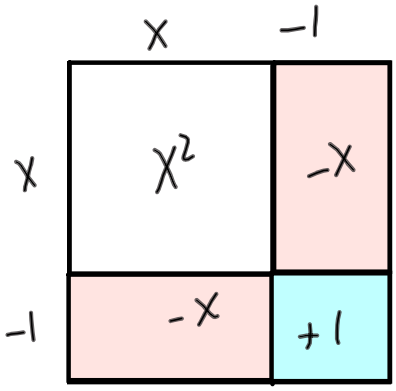
$2x = -4$

$x = -2$ ✓

$y = (-2)^2 + 4(-2) - 6$
$= 4 - 8 - 6$
$= -10$ ✓

4. The graph of $y = a(x - p)^2 + q$ has a minimum point (p, q).
By completing the square, find the minimum point of each of the following quadratic equations:

(ii) $3x^2 - 6x - 1 = 0$



$= 3\left[x^2 - 2x - \frac{1}{3}\right]$

$= 3\left[x^2 - 2x + 1 - 1 - \frac{1}{3}\right]$

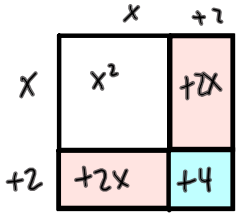
$= 3\left[(x-1)^2 - \frac{4}{3}\right]$

$= 3(x-1)^2 - 4$

min pt. = $(+1, -4)$

9. If $f(x) = x^2 + 4x + 7$, find
(i) the smallest possible value of $f(x)$
(ii) the value of x at which this smallest value occurs
(iii) the greatest possible value of $\frac{1}{(x^2 + 4x + 7)}$.

(i) min. value? Complete square



$x^2 + 4x + 7 = x^2 + 4x + 4 - 4 + 7 = (x+2)^2 + 3$

$\Rightarrow$ min. value $= 3$

(ii) this occurs $x = -2$

(iii) max. value of $\frac{1}{f(x)}$ happens when $f(x) = 3$

$\Rightarrow \frac{1}{3}$ is greatest value