10. Given that $a^2 + b^2 \geqslant 2ab$, deduce an expression for (i) $a^2 + c^2$ and (ii) $c^2 + d^2$.
(iii) Use these results to prove that

$$a^2 + b^2 + c^2 \geqslant ab + bc + ca \text{ for all real values of } a, b \text{ and } c.$$

If $a^2 + b^2 \geqslant 2ab$ (i) $\quad a^2 + c^2 \geq 2ac$

(ii) $\quad c^2 + d^2 \geq 2cd$

(iii)   * there seems to be a mistake in the question here! part (iii) does not include the letter "d". So result (ii) cannot be used to prove part (iii).

Proof of part (iii)

$$a^2 + b^2 \geq 2ab$$
$$a^2 + c^2 \geq 2ac$$
$$b^2 + c^2 \geq 2bc$$

Add LHS & RHS $\Rightarrow$ $2a^2 + 2b^2 + 2c^2 \geq 2ab + 2ac + 2bc$

$\div 2$ $\Rightarrow$ $a^2 + b^2 + c^2 \geq ab + ac + bc$

QED