Algebra 1

Core Revision Questions

p.41

Q1

(i)

$$\frac{12m^2n^3}{(6m^4n^5)^2} = \frac{{}^{1}\cancel{12}\,\cancel{m^2}\,\cancel{n^3}}{3\cancel{36}\,m^{\cancel{8}6}\,n^{\cancel{10}7}} = \frac{1}{3m^6n^7}$$

(ii)

$$\frac{3+\frac{1}{x}}{\frac{5}{x}+4} = \frac{3x+1}{5+4x}$$

(iii) $$\frac{2+\frac{x}{2}}{x^2-16} = \frac{\cancel{4+x}}{2\cancel{(x+4)}(x-4)}$$

$$= \frac{1}{2x-8}$$

Q2
(i) $y = x+4$

$5y + 2x = 6$

using substitution method (for change!)

$5(x+4) + 2x = 6$

$5x + 20 + 2x = 6$

$7x = -14$

$x = -2$

$y = -2 + 4 = 2$

Q2
(ii)

$$3x + y = 7 \Rightarrow y = 7 - 3x$$
$$x^2 + y^2 = 13$$

$$x^2 + (7-3x)^2 = 13$$
$$x^2 + 49 - 42x + 9x^2 = 13$$
$$10x^2 - 42x + 36 = 0$$
$$5x^2 - 21x + 18 = 0$$
$$(5x - 6)(x - 3) = 0$$
$$x = \frac{6}{5}, 3$$

$$y = 7 - 3\left(\frac{6}{5}\right)$$
$$= \frac{35 - 18}{5}$$
$$= \frac{17}{5} = 3\frac{2}{5}$$
pt $\left(1\frac{1}{5}, 3\frac{2}{5}\right)$

$$y = 7 - 3(3) = -2$$
pt $(3, -2)$

Q.3

$$\begin{array}{r|l} & x^2 + 2x - 1 \\ \hline x-3 & \cancel{x^3} - x^2 - 7x + 3 \\ & \cancel{\mp x^3} \pm 3x^2 \\ \hline & \cancel{+2x^2} - 7x \\ & \cancel{\mp 2x^2} \pm 6x \\ \hline & \cancel{-x} + \cancel{3} \\ & \cancel{\mp x} \mp \cancel{3} \\ \hline & 0 \end{array}$$

Q.4

$$\begin{array}{r|l} & 3x^3 + 6x^2 + 3x + 33 \\ \hline x-2 & 3x^4 + 0x^3 - 9x^2 + 27x - 66 \\ & \mp 3x^4 \pm 6x^3 \\ \hline & 6x^3 - 9x^2 \\ & \mp 6x^3 \pm 12x^2 \\ \hline & 3x^2 + 27x \\ & \mp 3x^2 \pm 6x \\ \hline & 33x - 66 \\ & \mp 33x \pm 66 \\ \hline \end{array}$$

Q.5 Solve

$$x^4 - 9x^2 = 0$$

$$x^2(x^2 - 9) = 0$$

$$(x^2)(x-3)(x+3) = 0$$

$x^2 = 0$	$x - 3 = 0$	$x + 3 = 0$
$x = 0$	$x = 3$	$x = -3$

Q5
(ii) Solve

$$(2x-1)^3(2-x)=0$$

$(a+b)^3$
$(a-b)^3$

$$(2x-1)(\underline{4x^2+2x+1})(2-x)=0$$

$2x-1=0$

$x=\frac{1}{2}$ ✓

$a=4$
$b=2$
$c=1$

$$x=\frac{-2\pm\sqrt{4-16}}{8}$$

PROBLEM
NO REAL
SOLUTION

$2-x=0$

$2=x$ ✓

Q6

$$4x^2+20x+k$$

is a perfect square
$k=?$

$$(2x+5)(2x+5)$$

$10x$

$10x$

$$4x^2+20x+\underline{25}$$

$$k=25$$

Q7

(i)

$$(3-\sqrt{2})^2 = a - b\sqrt{2}$$

$$9 - 6\sqrt{2} + 2 = a - b\sqrt{2}$$

$$11 - 6\sqrt{2} = a - b\sqrt{2}$$

$$a = 11 \quad , \quad b = 6$$

Q7 *NOT READY FOR THIS YET!

(ii)

$$\frac{(1-\sqrt{2})}{(1+\sqrt{2})} = a\sqrt{2} - b$$

TRICK
MULTIPLY ABOVE AND BELOW BY THE CONJUGATE OF BOTTOM!

The Conjugate of $1+\sqrt{2} \Rightarrow 1-\sqrt{2}$

$$\frac{(1-\sqrt{2})(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{1-2\sqrt{2}+2}{1-2} = \frac{3-2\sqrt{2}}{-1}$$

DIFF OF 2 SQUARES

$$= -3 + 2\sqrt{2}$$

Since

$$-3 + 2\sqrt{2} = a\sqrt{2} - b$$

$$\Rightarrow \quad a = 2 \quad , \quad b = 3$$

8. Factorise $x^3 - 27$.

Difference of 2 cubes

$(x-3)(x^2+3x+9)$

9. If $p(x-q)^2 + r = 2x^2 - 12x + 5$ for all values of x, find the values of p, q and r.

$p(x^2 - 2qx + q^2) + r = \text{RHS}$

$px^2 - 2qpx + pq^2 + r = 2x^2 - 12x + 5$

$\Rightarrow \quad p = 2$	$-2qp = -12$	$pq^2 + r = 5$
	$qp = 6$	$(2)(3)^2 + r = 5$
	$q(2) = 6$	$18 + r = 5$
	$q = 3$	$r = -13$

10. Solve the simultaneous equations
① $3x + 5y - z = -3$
② $2x + y - 3z = -9$
③ $x + 3y + 2z = 7.$

$$3x + 5y - z = -3$$
③ by -3 = $$-3x - 9y - 6z = -21$$
$$-4y - 7z = -24$$
④ $$4y + 7z = 24$$

$$2x + y - 3z = -9$$
③ by -2 = $$-2x - 6y - 4z = -14$$
⑤ $$-5y - 7z = -23$$

$$5y + 7z = 24$$
$$-4y - 7z = -23$$
$$y = 1$$

$$5(1) + 7z = 24$$
$$7z = 19$$
$$z = \frac{19}{7} = 2\tfrac{5}{7}$$

$$x + 3(1) + 2\left(\frac{19}{7}\right) = 7$$
$$x = -\frac{10}{7}$$

11. Simplify $(b + 1)^3 - (b - 1)^3$.

Difference of 2 Cubes

$$= \left((\cancel{b}+1) - (\cancel{b}-1)\right)\left((b+1)^2 + (b+1)(b-1) + (b-1)^2\right)$$
$$= (2)\left(b^2 + \cancel{2b} + 1 + b^2 - \cancel{1} + b^2 - \cancel{2b} + \cancel{1}\right)$$
$$= 2(3b^2 + 1)$$
$$= 6b^2 + 1$$

12. Find the rule for each of the following quadratic patterns.

(i) 3, 12, 27, 48, 75 ...

(ii) 5, 20, 45, 80, 125 ...

(iii) 0.5, 2, 4.5, 8, 12.5 ...

PATTERN	3 = b	12	27	48	75
1st DIFF.		9	15	21	27
2nd DIFF.			6 = 2a	6	6

$\Rightarrow a = 3$

Rule : $ax^2 + b$

$\Rightarrow 3x^2 + 3$

Check: $f(3) = 3(3)^2 + 3 = 30$ ✗

Rule doesn't work!

	3	12	27	48	75
$-3x^2$	-0	-3	-12	-27	-48
	3	9	15		
		6	6	6	6

LINEAR $ax + b$

$= 6x + 3$

ADD $3x^2$ to LINEAR $\Rightarrow$ Rule is $3x^2 + 6x + 3$

Check: $f(3) = 3(3)^2 + 6(3) + 3 = 48$ ✓

12. Find the rule for each of the following quadratic patterns.

(i) 3, 12, 27, 48, 75 ...
(ii) 5, 20, 45, 80, 125 ...
(iii) 0.5, 2, 4.5, 8, 12.5 ...

PATTERN	5 =b	20	45	80	125
1st DIFF.		15	25	35	45
2ND DIFF.			10 =2a	10	10

$a=5$

Rule: $ax^2+b \quad = \quad 5x^2+5$

Check: $f(0) = 0$ ✓
$f(3) = 5(3)^2+5 = 50$ X

Rule doesn't work.

12. Find the rule for each of the following quadratic patterns.

(i) 3, 12, 27, 48, 75 ...
(ii) 5, 20, 45, 80, 125 ...
(iii) 0.5, 2, 4.5, 8, 12.5 ...

Pattern	0.5 =b	2	4.5	8	12.5
1st Diff.		1.5	2.5	3.5	4.5
2nd Diff			1 =2a	1	1

QUADRATIC

$a=\frac{1}{2}$

Rule: $ax^2+b \quad = \quad \frac{1}{2}x^2+\frac{1}{2}$

Check: $f(0) = \frac{1}{2}(0)^2+\frac{1}{2} = \frac{1}{2}$ ✓
$f(4) = \frac{1}{2}(4)^2+\frac{1}{2} = \frac{1}{2}(16)+\frac{1}{2} = 8.5$ X

So Rule doesn't work

	5	20	45	80	125
$-5x^2$	-0	-5	-20	-45	-80
	5	15	25	35	45
		10	10	10	10

LINEAR $ax+b$
$= 10x+5$

LINEAR + $5x^2$ $\Rightarrow$ Rule is $5x^2+10x+5$

Check $f(4) = 5(4)^2 + 10(4) + 5 = 125$ ✓

Q12 (iii) Continued : We know its quadratic

	0.5	2	4.5	8	12.5
$-\frac{1}{2}x^2$ term	-0	-0.5	-2	-4.5	-8
	0.5	1.5	2.5	3.5	4.5
		1	1	1	1

LINEAR
$ax+b$
$= 1x + \frac{1}{2}$

ADD $\frac{1}{2}x^2$ to LINEAR $\Rightarrow$ Rule is $\frac{1}{2}x^2 + x + \frac{1}{2}$

check: $f(3) = \frac{1}{2}(3)^2 + 3 + \frac{1}{2} = 8$ ✓

13. Find the rule for the pattern 6, 12, 20, 30, 42 using first and second differences. Hence find the 100th term of this pattern.

1st Difference	6	8	10	12	QUADRATIC
2nd Difference		2	2	2	

$b = 6$, $2a = 2 \Rightarrow a = 1$

Rule: $ax^2 + b = x^2 + 6$

$f(0) = 0^2 + 6 = 6$ ✓ The sequence 6, 7, 10, 15 is not the required pattern!

$f(1) = 1^2 + 6 = 7$ x

$f(2) = 2^2 + 6 = 10$ x

$f(3) = 3^2 + 6 = 15$ x

	6	12	20	30	42	
Subtract x^2 term	−0	−1	−4	−9	−16	
	6	11	16	21	26	← LINEAR $a = 5$
		5	5	5	5	$b = 6$

LINEAR RULE : $5x + 6$

ADD x^2 TERM : x^2

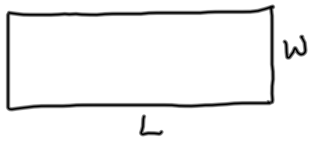
QUADRATIC RULE: $x^2 + 5x + 6$

Check $f(0) = 6$ ✓

$f(4) = (4)^2 + 5(4) + 6 = 42$ ✓

$\Rightarrow$ Term 100 $= f(99) = (99)^2 + 5(99) + 6 = 10,302$

14. Three times the width of a certain rectangle exceeds twice the length by 3 cm.
Four times the length is 12 cm more than its perimeter.
Find the dimensions of the rectangle.



$3W - 2L = 3$

Perimeter $= 2L + 2W$

$4L - (2L + 2W) = 12$

$2L - 2W = 12$

$$\begin{aligned} -2W + 2L &= 12 \\ 3W - 2L &= 3 \\ \hline W \quad &= 15 \text{ cm} \end{aligned}$$

$2L = 12 + 2W$

$2L = 12 + 2(15) = 42$

$L = 21$ cm

15. The formula for a spherical mirror of radius r cm is given by $\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$, where u cm is the object distance and v cm is the image distance to the mirror.

The magnification in the mirror is given by $m = \frac{v - r}{r - u}$.

(i) Find r in terms of u and v.

(ii) Find m in terms of v and u only.

(i)

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

Single fraction

$$\frac{v + u}{uv} = \frac{2}{r}$$

invert both sides

$$\frac{r}{2} = \frac{uv}{v + u}$$

multiply by 2

$$r = \frac{2uv}{v + u}$$

(ii)

Sub in r expression

$$m = \frac{v - \left(\frac{2uv}{v+u}\right)}{\left(\frac{2uv}{v+u}\right) - u}$$

multiply by $(v+u)$

$$m = \frac{v(v+u) - 2uv}{2uv - u(v+u)} = \frac{v^2 + uv - 2uv}{2uv - uv - u^2}$$

$$m = \frac{v^2 - uv}{uv - u^2} = \frac{v(1-u)}{u(v-1)}$$