Section 1.1 Polynomial expressions

4. Simplify each of the following.

(i) $3x^2 - 6x + 7 + 5x^2 + 2x - 9$

$= 7x^2 - 4x - 2$

Remember...

You can only add or subtract like terms

6. Expand each of the following.

(iii) $(3x - 2)(x + 3)$

$= 3x(x+3) - 2(x+3)$

$= 3x^2 + 9x - 2x - 6$

$= 3x^2 + 7x - 6$

Remember...

'Expand' means 'multiply'

10. If $25x^2 + tx + 4$ is a perfect square for all values of x, find the value of t.

Remember...

A perfect square has this shape

$(a+b)^2 = a^2 + 2ab + b^2$

$(5x+2)^2 = 25x^2 + 20x + 4$

$\Rightarrow t = 20$

23. Simplify each of the following:

(i) $\dfrac{2x^2 + 5x - 3}{2x - 1} = \dfrac{\cancel{(2x-1)}(x+3)}{\cancel{(2x-1)}} = x+3$

Section 1.2 Polynomial functions, an introduction

11. The volume of a cone, $V(r, h)$, is given by the formula $V(r, h) = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the perpendicular height of the cone. Find

(i) the volume, in terms of π, of a cone with height 21 cm and radius 14 cm

(ii) the volume of a cone, in terms of r and π, if the cone has the same height as the radius r

(iii) the volume of a cone, in terms of h and π, if the radius of the base is twice the height h.

(i) $h = 21$ cm $\quad r = 14$ cm

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (14)^2 (21) = 1372\pi \text{ cm}^3$$

(ii) $h = r$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (r)^2 (r) = \frac{1}{3}\pi r^3 \text{ cm}^3$$

(iii) $r = 2h$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (2h)^2 h = \frac{1}{3}\pi\, 4h^2 h = \frac{4}{3}\pi h^3$$

Section 1.3 Factorising algebraic expressions

Using the highest common factor, factorise each of the following:

7. $2a^2b - 4ab^2 + 12abc$

$= 2ab(a - 2b + 6c)$

Factorise each of the following by grouping terms.

12. $\underbrace{2c^2 - 4cd} + \underbrace{c - 2d}$

$= 2c(c - 2d) + 1(c - 2d)$

$= (2c + 1)(c - 2d)$

Using the difference of two squares, factorise the following:

23. $1 - 36x^2$

$= (1 + 6x)(1 + 6x)$

Remember...

Difference of 2 Squares

$a^2 - b^2 = (a+b)(a-b)$

Factorise each of the following quadratic expressions:

38. $2x^2 - 7x + 3$

$-1x$

$= (2x - 1)(x - 3)$

$-6x$

$-6x - 1x = -7x$ ✓

Factorise each of the following quadratic expressions:

50. $12x^2 + 17xy - 5y^2$

FACTORS TO CONSIDER

$12x^2$	$-5y^2$
$(12x)(1x)$	$(-5y)(y)$
$(6x)(2x)$	$(5y)(-y)$
$(3x)(4x)$	

$-3xy$

$$(4x - y)(3x + 5y)$$

$+20xy$

$+20xy - 3xy = 17xy$ ✓

FACTORISE

53. (i) $27x^3 - y^3$

$$= (3x - y)(9x^2 + 3xy + y^2)$$

Remember...

Difference of 2 cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Section 1.4 Simplifying algebraic fractions

2. Express each of the following as a single fraction:

(h) $\dfrac{3x+5}{6} - \dfrac{2x+3}{4} - \dfrac{1}{12}$

LCD $=12$

$$= \frac{2(3x+5) - 3(2x+3) - 1(1)}{12}$$

$$= \frac{\cancel{6x} + \cancel{10} - \cancel{6x} - \cancel{9} - \cancel{1}}{12} = \frac{0}{12} = 0$$

3. By factorising the numerator and the denominator fully, simplify each of the following.

(v) $\dfrac{2}{a+4} - \dfrac{a+2}{a^2-9}$

$$= \frac{2(a^2-9) - (a+4)(a+2)}{(a+4)(a^2-9)}$$

← note the denominator is ok like this

Remember...

You can use the 'bow-tie' method to add/subtract fractions

$$= \frac{2a^2 - 18 - [a^2 + 2a + 4a + 8]}{(a+4)(a^2-9)}$$

$$= \frac{2a^2 - 18 - a^2 - 6a - 8}{(a+4)(a^2-9)} = \frac{a^2 - 6a - 26}{(a+4)(a^2-9)}$$

* note: answer in book is incorrect

7. Simplify (iii) $\dfrac{x+y}{\dfrac{1}{x}+\dfrac{1}{y}}$

multiply each part by xy

$$= \frac{x(xy)+y(xy)}{\frac{1}{x}(xy)+\frac{1}{y}(xy)} = \frac{xy(x+y)}{y+x}$$

$$= xy$$

11. Simplify each of the following.

(i) $\dfrac{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}{1+\dfrac{a-b}{a+b}}$

multiply each part by (a+b)(a-b)

Remember...

Difference of 2 Squares

$x^2-y^2=(x+y)(x-y)$

$$= \frac{\dfrac{(a+b)(a+b)\cancel{(a-b)}}{\cancel{(a-b)}}-\dfrac{(a-b)\cancel{(a+b)}(a-b)}{\cancel{(a+b)}}}{1(a+b)(a-b)+\dfrac{(a-b)\cancel{(a+b)}(a-b)}{\cancel{(a+b)}}}$$

$$= \frac{(a+b)^2-(a-b)^2}{(a-b)(a+\cancel{b}+a-\cancel{b})} = \frac{(a+\cancel{b}+a-\cancel{b})(\cancel{a}+b-\cancel{a}+b)}{(a-b)(2a)}$$

HCF

$$= \frac{(2\cancel{a})(2b)}{(2\cancel{a})(a-b)} = \frac{2b}{a-b}$$

Section 1.5 Algebraic identities

6. Find the values of a and b if $(2x + a)^2 = 4x^2 + 12x + b$ for all x.

Remember...

if $ax^2 + bx + c = dx^2 + ex + f$

then: $a = d$
$b = e$
$c = f$

$(2x+3)^2 = 4x^2 + 12x + 9$

$\Rightarrow \quad b = 9$

21. If $(x - 2)^2$ is a factor of $x^3 + px + q$, find the value of p and the value of q.

$(x-2)^2 = x^2 - 4x + 4$

$$
\begin{array}{r|l}
 & x + 4 \\
x^2 - 4x + 4 & x^3 + 0x^2 + px + q \\
 & \mp x^3 \pm 4x^2 \mp 4x \\
 & 4x^2 + (p-4)x + q \\
 & \mp 4x^2 \pm 16x \mp 16 \\
 & (p+12)x + (q-16) = 0x + 0
\end{array}
$$

$p + 12 = 0$, $q - 16 = 0$

$p = -12 \qquad q = 16$

Remember...

A factor divides leaving no remainder

27. If $x^2 + ax + b$ is a factor of $x^3 - k$, show that (i) $a^3 = k$ (ii) $b^3 = k^2$.

> Remember...
>
> A factor divides leaving no remainder

$$
\begin{array}{r l}
 & \quad x - a \\
x^2+ax+b\,\big) & x^3 + 0x^2 + 0x - k \\
 & \mp x^3 \mp ax^2 \mp bx \\
\hline
 & \qquad -ax^2 - bx + k \\
 & \qquad \pm ax^2 \pm a^2x \pm ab \\
\hline
 & \qquad \underbrace{(a^2-b)}_{=0}x + \underbrace{(ab-k)}_{=0} = 0x + 0
\end{array}
$$

$$
\begin{array}{l|l|l}
a^2 - b = 0 & ab - k = 0 & k = ab \\
a^2 = b & ab = k & k^2 = a^2b^2 \\
 & a(a^2) = k & k^2 = (b)(b^2) \\
 & a^3 = k \checkmark & k^2 = b^3 \checkmark
\end{array}
$$

Section 1.6 Manipulating formulae

7. In each of the following, express a in terms of the other variables:

(i) $\dfrac{x}{y} = \dfrac{a+b}{a-b}$

> Remember...
>
> You must do the same thing to both sides

multiply by LCD y(a - b)

$x(a-b) = y(a+b)$ expand

$ax - bx = ay + by$ bring 'a' terms to LHS

$ax - ay = bx + by$ factorise

$a(x-y) = b(x+y)$ divide by (x - y)

$$a = \frac{b(x+y)}{(x-y)}$$

Section 1.6 Manipulating formulae

7. In each of the following, express a in terms of the other variables:

> Remember...
> You must do the same thing to both sides

divide by c (ii) $b\cancel{c} - a\cancel{c} = a\cancel{c}.$

bring 'a' terms to RHS $b = 2a$

divide by 2 $\frac{b}{2} = a$

10. Write c in terms of the other variables in each of the following.

(i) $d = \sqrt{\frac{a-b}{ac}}$ square both sides

$d^2 = \frac{a-b}{ac}$ multiply by c and divide by d^2

$c = \frac{a-b}{ad^2}$

10. Write c in terms of the other variables in each of the following.

(ii) $b = \dfrac{2c - 1}{c - 1}$ multiply by c -1

$b(c-1) = 2c - 1$ expand

$bc - b = 2c - 1$ c terms to LHS

$bc - 2c = b - 1$ factorise

$c(b-2) = b - 1$ divide by b - 2

$c = \dfrac{b-1}{b+2}$

Section 1.7 Algebraic patterns, an introduction

1. Examine each of the following patterns of numbers and determine if the pattern has a linear or quadratic relationship.

(a) 4, 7, 10, 13, 16, …

x	0	1	2	3	4
f(x)	4	7	10	13	16
1st Difference		3	3	3	3
2nd Difference			0	0	0

This is Linear

Remember...
If 2nd Difference is constant then it is a Quadratic pattern

Remember...
If 1st Difference is constant then it is a Linear pattern

1. Examine each of the following patterns of numbers and determine if the pattern has a linear or quadratic relationship.

(i) 0, 3, 12, 27, 48, ...

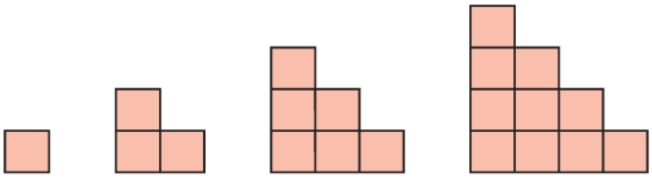
Remember...
If 2nd Difference is constant then it is a Quadratic pattern

Remember...
If 1st Difference is constant then it is a Linear pattern

x	0	1	2	3	4
f(x)	0	3	12	27	48
1st Difference		3	9	15	21
2nd Difference			6	6	6

This is Quadratic

1. By converting the following designs into a number pattern, write down a rule for the pattern. Use the rule to find out how many bricks are needed to build the 49th design.



Remember...
If 2nd Difference is constant then it is a Quadratic pattern

x	0	1	2	3
f(x)	1 = b	3	6	10
1st Difference		2	3	4
2nd Difference			1 = 2a	1

This is Quadratic

$2a = 1 \Rightarrow a = \frac{1}{2}$

and $b = 1$

Does $f(x) = ax^2 + b$?

So Does $f(x) = \frac{1}{2}x^2 + 1$?

Check $f(3) = \frac{1}{2}(3)^2 + 1 = \frac{9}{2} + 1 = 5\frac{1}{2} \neq 10$

This means that we are missing the x term.

We know x^2 term is $\frac{1}{2}x^2$ →

x	0	1	2	3
f(x)	1	3	6	10
$-\frac{1}{2}x^2$	-0	$-\frac{1}{2}$	-2	$-4\frac{1}{2}$
Linear	1 $=b$	2.5	4	5.5
1st Difference		1.5 $=a$	1.5	1.5

$\Rightarrow$ Linear pattern is $ax+b = \frac{3}{2}x + 1$

$\Rightarrow$ $f(x) = -\frac{1}{2}x^2 + \frac{3}{2}x + 1$

Section 1.8 Solving equations

4. Solve (iii) $\dfrac{x-3}{4} = \dfrac{x-2}{5}$ multiply by the LCM = 20

$5(x-3) = 4(x-2)$

$5x - 15 = 4x - 8$

$x = 7$

6. Find the value of the unknown in each of the following equations:

(iv) $\dfrac{3r-2}{5} - \dfrac{2r-3}{4} = \dfrac{1}{2}$ multiply by the LCM = 20

$4(3r-2) - 5(2r-3) = 10(1)$

$12r - 8 - 10r + 15 = 10$

$2r + 7 = 10$

$2r = 3$

$r = \frac{3}{2}$

7. Solve each of the following:

(ii) $\frac{2}{3}(x-1) - \frac{1}{5}(x-3) = x + 1$ multiply by the LCM = 15

$10(x-1) - 3(x-3) = 15(x+1)$

$10x - 10 - 3x + 9 = 15x + 15$

$7x - 1 = 15x + 15$

$-16 = 8x$

$x = -2$

Section 1.9 Solving simultaneous linear equations

2. Solve

multiply by 5   divide by 2

(iii) $\frac{4x-2}{5} = \frac{\cancel{8}4y}{\cancel{10}5}$ $\Rightarrow 4x - 2 = 4y \Rightarrow 4x - 4y = 2 \Rightarrow 2x - 2y = 1$ ①

$18x - 20y = 4 \Rightarrow 9x - 10y = 2$ ②

divide by 2

① ×5 $\quad 10x - 10y = 5$

② ×−1 $\quad -9x + 10y = -2$

$x = 3$

$2(3) - 2y = 1$

$-2y = 1$

$6 - 2y = 1$

$-2y = -5$

$y = 5/2$

5. Solve the following equations with three unknowns.

(iii) $2x + y - z = 9$ ①
$x + 2y + z = 6$ ②
$3x - y + 2z = 17$ ③

① + ② $\quad 2x + y - z = 9$

$x + 2y + z = 6$

$3x + 3y = 15$

$x + y = 5$ ④

2① + 3 $\quad 4x + 2y - 2z = 18$

$3x - y + 2z = 17$

$7x + y = 35$ ⑤

⑤ − ④ $\quad 7x + y = 35$

$-x - y = -5$

$6x = 30$

$x = 5$

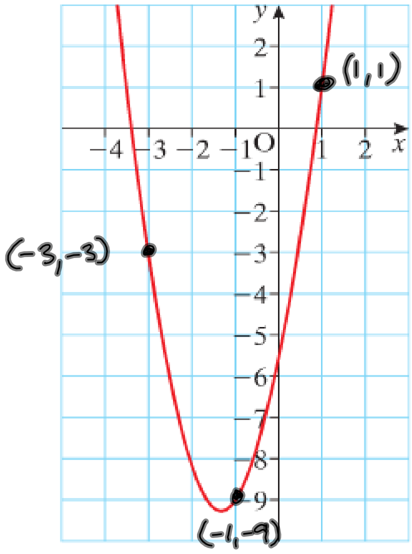
Sub into ④ $\quad 5 + y = 5$

$y = 0$

Sub into ② $\quad 5 + 2(0) + z = 6$

$z = 1$

9. A curve of the form $f(x) = y = ax^2 + bx + c$ is drawn as shown.
By picking any three points on the curve, form three equations connecting the coefficients a, b and c and hence solve to find $f(x)$.



Sub in (1,1) $a(1)^2 + b(1) + c = 1$

$a + b + c = 1$ ①

Sub. in (−3,−3) $a(-3)^2 + b(-3) + c = -3$

$9a - 3b + c = -3$ ②

Sub in (−1,−9) $a(-1)^2 + b(-1) + c = -9$

$a - b + c = -9$ ③

①−③

$$\begin{array}{r} a + b + c = 1 \\ -a + b - c = 9 \\ \hline 2b \quad = 10 \end{array}$$

$b = 5$ ④

②−①

$$\begin{array}{r} 9a - 3b + c = -3 \\ -a - b - c = -1 \\ \hline 8a - 4b = -4 \end{array}$$

$2a - b = -1$

Sub in ④

$2a - 5 = -1$

$2a = 4$

$a = 2$

$a + b + c = 1$

Sub in values

$2 + 5 + c = 1$

$c = -6$

10. 44,000 people attended a match in Croke Park. The two ticket prices on the day were €30 and €20. The total receipts for the game came to €1.2 million.
How many people paid the higher ticket price?

let x = no. people who paid higher price

let y = no. people who paid the lower price

$$x + y = 44{,}000$$

$$30x + 20y = 1{,}200{,}000$$

$$3x + 2y = 120{,}000$$

$$\begin{array}{r} 3x + 2y = 120{,}000 \\ -2x - 2y = -88{,}000 \\ \hline x \qquad = 32{,}000 \end{array}$$