Revision

The Line and

The Circle

14 Jan 2013

2.2 Co-ordinate geometry

- use slopes to show that two lines are
  - parallel
  - perpendicular
- recognise the fact that the relationship $ax + by + c = 0$ is linear
- solve problems involving slopes of lines

- calculate the area of a triangle
- recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the $x$ and $y$ co-ordinates of points on a circle centre $(h, k)$ and radius $r$
- solve problems involving a line and a circle with centre $(0, 0)$

- solve problems involving
  - the perpendicular distance from a point to a line
  - the angle between two lines
- divide a line segment internally in a given ratio $m:n$
- recognise that $x^2+y^2 +2gx+2fy+c = 0$ represents the relationship between the $x$ and $y$ co-ordinates of points on a circle centre $(-g,-f)$ and radius $r$ where $r = \sqrt{(g^2+f^2 – c)}$

- solve problems involving a line and a circle
Coordinate geometry of the circle assumes knowledge of coordinate geometry of the line.

Coordinate Geometry of The Line
from Junior Cent

| Distance between 2 points | \[|ab| = \sqrt{(x_b-x_a)^2 + (y_b-y_a)^2}\] |
|---------------------------|--------------------------------------------------|
| Midpoint (average point)  | \[\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\] |
| Slope (given 2 points)    | \[m = \frac{y_b-y_a}{x_b-x_a}\] |
| Slope (given graph)       | \[m = \frac{\text{rise}}{\text{run}}\] |

Slope from equation of line
slope (given: \(y = mx + c\))

\[m = m_1\]

slope (given \(ax + by + c = 0\))

\[m = -\frac{a}{b}\]

parallel slopes
\[\Rightarrow m_1 = m_2\]

perpendicular slopes
\[\Rightarrow m_1 \times m_2 = -1\]

\[\frac{a}{b} \neq -\frac{b}{a}\] if \[m_1 = \frac{a}{b} \Rightarrow m_2 = -\frac{b}{a}\]

Equation of a line
\[y - y_1 = m(x - x_1)\]

Graph line (given \(ax + by + c = 0\))

Graph line (given: \(y = mx + c\))

Point on a line

Point of intersection of 2 lines

Image of \(a\) under...
axial symmetry 
\[S_x \rightarrow a\]
central symmetry 
\[S_y \rightarrow b\]
transformation 
\[S \rightarrow c\]

Transformations

Let \(x = 0\), find \(y\)
Let \(y = 0\), find \(x\) ....

\[m = \frac{\text{rise}}{\text{run}}\]
\[c = \text{y-intercept}\]

If \((x_1, y_1)\) is on the line \(ax + by + c = 0\) then \(ax_1 + by_1 + c = 0\)

Solve simultaneous equations
Revision of Coordinate Geometry

January 12, 2013

Coordinate Geometry of The Line

Area of a circle with vertex (0, 0)

\[ A = \frac{1}{2} \left| x_1 y_2 - x_2 y_1 \right| \]

⇒ transform triangle so it has a vertex at (0, 0)

The angle between 2 lines

\[ \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} \]

The perpendicular distance from a point to a line

\[ d = \left| \frac{a x_1 + b y_1 + c}{\sqrt{a^2 + b^2}} \right| \]

Internally divide a line segment in the ratio \( m:n \)

The Circle

Diameter, radius

Centre is midpoint of diameter

2r = d

c is midpoint of [ab]

Area & Circumference

Area = \( \pi r^2 \)

Circumference = 2\( \pi r \)

Area of sector = \( \frac{B}{360} \pi r^2 \)

Arc length = \( \frac{B}{360} \cdot 2\pi r \)

Angles standing on same arc

Angle of 90° stands on diameter

Perpendicular line from centre through chord bisects chord

Tangent is perpendicular to line [cd]
The Circle

**Leaving Cert. Ordinary Level**

1. **Equation of a circle**
   - Centre $(a, b)$, Radius $r$
   - $(x-a)^2 + (y-b)^2 = r^2$

2. **Points in, on, outside a circle**
   - Sub. point into the equation
   - Point inside circle: $|cp| < r$
   - Point on circle: $|cp| = r$
   - Point outside circle: $|cp| > r$

3. **Intersection of a line and a circle**
   - Solve simultaneous equations
   - Rewrite linear
   - Sub into circle
   - Solve quadratic
   - Sub solutions into linear

4. **Circle intersecting axes**
   - Intersects x-axis where $y=0$
   - Intersects y-axis where $x=0$
   - $\Rightarrow$ only 1 point of intersection
   - Also distance from $c$ to $T$ is $r$

5. **Proving a line is a tangent to a circle**

**Leaving Cert. Higher Level**

6. **General equation of a circle**
   - Centre $= (-g, -f)$
   - $x^2 + y^2 + 2gx + 2fy + c = 0$
   - Radius $= \sqrt{g^2 + f^2 - c}$

7. **Touching circles**
   - Touch externally
   - Touch internally
   - $d = r_1 + r_2$
   - $d = r_1 - r_2$

8. **Equation of tangent**
   - Find slope $[cf]$
   - Find slope $T$
   - $y - y_1 = m(x - x_1)$
   - Or use formula in log tables

9. **Common chord (or tangent)**
   - Circles $s_1$ and $s_2$ expressed in the form
   - $x^2 + y^2 + 2gx + 2fy + c = 0$
   - Then $s_1 - s_2 = L$

10. **Circles touching axes**
    - Touches x-axis
      - $g^2 = c$ and $r = |f|$
    - Touches y-axis
      - $f^2 = c$ and $r = |g|$