Revision

The Line and

The Circle

14 Jan 2013

2.2 Co-ordinate geometry	– use slopes to show that two lines are • parallel • perpendicular – recognise the fact that the relationship $ax + by + c = 0$ is linear – solve problems involving slopes of lines	– calculate the area of a triangle – recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle centre (h, k) and radius r – solve problems involving a line and a circle with centre (0, 0)	– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment internally in a given ratio m:n – recognise that $x^2+y^2+2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle centre $(-g,-f)$ and radius r where $r = \sqrt{(g^2+f^2 - c)}$ – solve problems involving a line and a circle

Coordinate geometry of the circle assumes knowledge of coordinate geometry of the line.

Coordinate Geometry of The Line

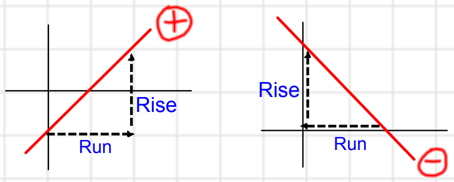
from Junior Cent

distance between 2 points: $|ab| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint (average point): $\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

slope (given 2 points): $m = \frac{y_2-y_1}{x_2-x_1}$

slope (given graph): $m = \frac{\text{Rise}}{\text{Run}}$



Slope from equation of line slope (given: y = mx + c): $m = m$

Slope from equation of line slope (given ax + by+ c = 0): $m = \frac{-a}{b}$

parallel slopes: $\Rightarrow m_1 = m_2$

perpendicular slopes: $\Rightarrow m_1 \times m_2 = -1$

$\frac{a}{b} \perp -\frac{b}{a}$ if $m_1 = \frac{a}{b} \Rightarrow m_2 = -\frac{b}{a}$

from Junior Cent

equation of a line: $y - y_1 = m(x - x_1)$

Graph line (given ax + by+ c = 0): let $x=0$, find y; let $y=0$, find x ....

Graph line (given: y = mx + c): $m = \frac{\text{Rise}}{\text{Run}}$ $c =$ y-intercept

point on a line: If (x_1, y_1) is on the line $ax + by + c = 0$ then $ax_1 + by_1 + c = 0$

point of intersection of 2 lines: Solve simultaneous equations

Transformations

Image of a under...

axial symmetry

$S_x \to d$

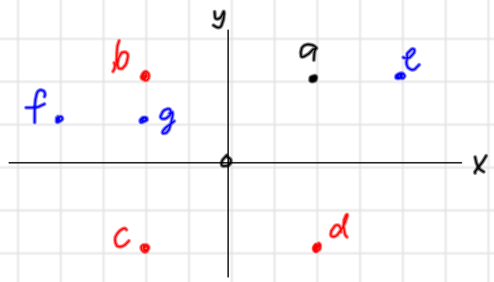
$S_y \to b$

central symmetry

$S_o \to c$

transformation

$\vec{fg} \to e$



Coordinate Geometry of The Line

Leaving Cert. Ordinary Level

Area of a circle with vertex (0, 0)

$$A = \frac{1}{2}\left| x_1 y_2 - x_2 y_1 \right|$$

Area of a circle (0, 0) not vertex

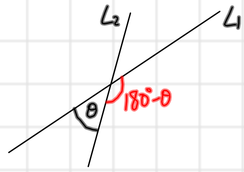
⇒ transform triangle so it has a vertex at (0, 0)

Coordinate Geometry of The Line

Leaving Cert. Higher Level

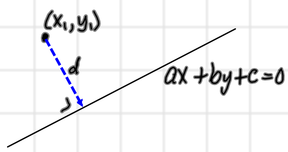
The angle between 2 lines

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

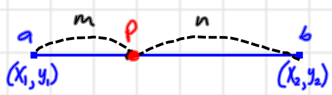


The perpendicular distance from a point to a line

$$d = \frac{\left| a x_1 + b y_1 + c \right|}{\sqrt{a^2 + b^2}}$$



Internally divide a line segment in the ratio m:n



can use ratios or formula

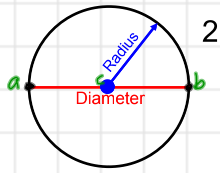
$$p = \left(\frac{m x_2 + n x_1}{m + n}, \frac{m y_2 + n y_1}{m + n} \right)$$

The Circle

Junior Cert. Geometry

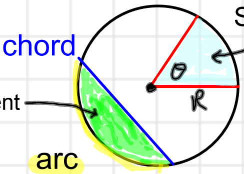
Diameter, radius

Centre is midpoint of diameter



$2r = d$

c is midpoint of [ab]



Area & Circumference

Area $= \pi R^2$

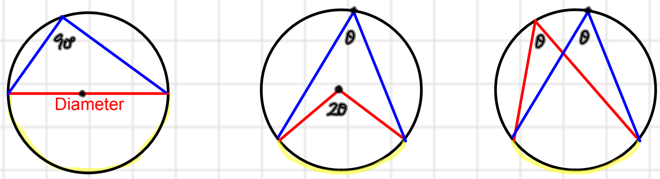
Circumference $= 2\pi R$

Area of sector $= \left(\frac{\theta}{360°}\right)\pi r^2$

Arc length $= \left(\frac{\theta}{360°}\right)2\pi R$

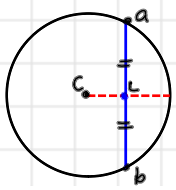
Angles standing on same arc

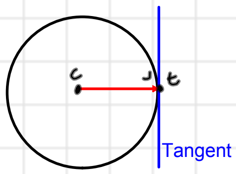
Angle of 90° stands on diameter



Perpendicular line from centre through chord bisects chord

Tangent is perpendicular to line [ct]





The Circle

Leaving Cert. Ordinary Level

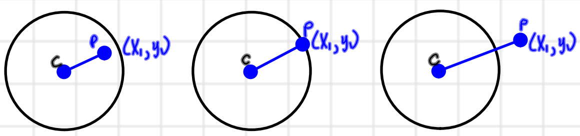
① Equation of a circle

Centre $(0,0)$, Radius r: $x^2 + y^2 = r^2$

Centre (h,k), Radius r: $(x-h)^2 + (y-k)^2 = r^2$

② Points in, on, outside a circle

Point inside circle | Point on circle | Point outside circle



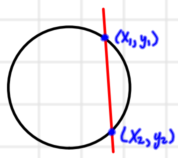
$|cp| < r$ $\quad$ $|cp| = r$ $\quad$ $|cp| > r$

Sub. point into the equation

$(x_1-h)^2+(y_1-k)^2 < r^2$ $\quad$ $(x_1-h)^2+(y_1-k)^2 = r^2$ $\quad$ $(x_1-h)^2+(y_1-k)^2 > r^2$

③ Intersection of a line a circle

Solve simultaneous equations



- rewrite linear
- sub into circle
- solve quadratic
- sub solutions into linear

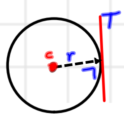
④ Circle intersecting axes

Intersects x-axis where y=0

Intersects y-axis where x=0

⑤ Proving a line is a tangent to a circle

⇒ only 1 point of intersection



also distance from c to T is r

The Circle

Leaving Cert. Higher Level

(6) General equation of a circle

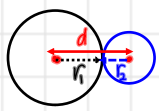
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $= (-g, -f)$

Radius $= \sqrt{g^2 + f^2 - c}$

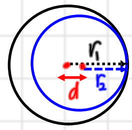
(7) Touching circles

Touch externally



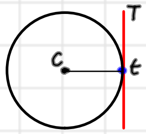
$d = r_1 + r_2$

Touch internally



$d = r_1 - r_2$

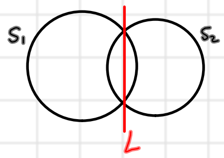
(8) Equation of tangent



- find slope [ct]
- find slope T
- $y - y_1 = m(x - x_1)$

or use formula in log tables

(9) Common chord (or tangent)



circles s_1 and s_2 expressed in the form

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

then $S_1 - S_2 = L$

(10) Circles touching axes

touches x-axis

$\Rightarrow g^2 = c$ and $r = |-f|$

touches y-axis

$\Rightarrow f^2 = c$ and $r = |-g|$