Financial Maths

Revision Exercise

(Core)

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1. A woman invests €1000 each year at 8% per annum, compound interest. Find the value of her investment after 5 years.

$A = P(1+i)^t$

Series

$$\overset{T_5}{(1000)(1\cdot08)^5} + \overset{T_4}{(1000)(1\cdot08)^4} + \ldots + \overset{T_1 = a}{(1000)(1\cdot08)^1}$$

$S_n = \dfrac{a(1-r^n)}{(1-r)}$

$$S_5 = \frac{1080(1-1\cdot08^5)}{1-1\cdot08} = €\ 6335\cdot93$$

2. €300 is invested each month for eight years.
Find the total value of the investment after eight years, assuming a constant rate of 6% per annum.

$r = \sqrt[12]{1+i} - 1$

8 year = 96 months

$A = P(1+i)^t$

Series

$S_n = \frac{a(1-r^n)}{1-r}$

$r = \sqrt[12]{1.06} - 1 = 0.0048675$

$\overset{T_{96}}{300(1.0048675)^{96}} + \ldots\ldots + \overset{T_1 = a}{300(1.004875)^1}$

$S_{96} = \frac{301.46(1 - 1.00487^{96})}{1 - 1.00487}$

$\approx$ €36,783.14 ✓

3. A car loan of €20 000 is to be repaid in 25 equal instalments.
If the effective interest rate is 2%, calculate the amount of each instalment, correct to the nearest euro.

$A = \frac{P(i)(1+i)^n}{(1+i)^n - 1}$

$i = 0.02$
$n = 25$
$P = €20{,}000$

$A = \frac{20000(0.02)(1.02^{25})}{1.02^{25} - 1}$

$=$ €1,024.41 ✓

4. Silvia is planning an overseas trip lasting 3 years and she estimates that she will need €600 per month for expenses.
How much money does she need to have saved to fund this trip?
Assume an average rate of interest of 4% over the period of the trip.

note: I have interpreted this 4% interest rate to be the 3 year equivalent rate, the answer given is corrected based on this assumption. The suggested solution in the book takes the view that the 4% is an AER.

$n = 12(3) = 36$

$A = \frac{P(i)(1+i)^n}{(1+i)^n - 1}$

$r = \sqrt[36]{1.04} - 1 = 0.00109$

$600 = \frac{P(0.00109)(1.00109^{36})}{(1.00109^{36} - 1)}$

$600 = P(0.0283)$

$P = \frac{600}{0.0283} = €21,201.41$ ✓

5. A credit card company offers clients an introductory interest rate on outstanding balances of 1.25% per month, and a regular rate of 2.5% per month after 1 year.
Find the equivalent interest rates (AER) per annum.

MER = R
AER = i

$1+i = (1+R)^{12}$

MER 1.25% → AER ?

$1 + i = (1.0125)^{12}$

$\Rightarrow i = 1.0125^{12} - 1 = 0.1607 = 16.1\%$ ✓

$1+i = (1+R)^{12}$

MER 2.5% → AER ?

$1+i = (1.025)^{12}$

$i = 1.025^{12} - 1 = 0.3448 \approx 34.5\%$ ✓

6. John makes savings of €200 a month for 5 months at an effective monthly rate of 0.75%.
Express these savings as a geometric series.
Write down the first term, the common ratio and an expression for the sum of the five terms.

$A = P(1+i)^t$

Series

$$\overset{T_5}{200(1\cdot0075)^5} + \ldots + \overset{T_1 = a}{200(1\cdot0075)^1}$$

$n = 5$

$a = €\ 201\cdot5$ Ratio, $R = 1\cdot0075$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_5 = \frac{201.5(1 - 1\cdot0075^5)}{1 - 1\cdot0075}$$

$$= €\ 1022\cdot73$$

7. Use the future value formula to find what €1600 would amount to if invested each year for 5 years at 6 % p.a. compound interest.

$F = P(1+i)^n$

Series, $F = \overset{T_5}{1600(1\cdot06)^5} + \ldots + \overset{T_1 = a}{1600(1\cdot06)^1}$

$a = 1696$, $r = 1\cdot06$, $n = 5$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_5 = F = \frac{1696(1 - 1\cdot06^5)}{1 - 1\cdot06}$$

$$= €\ 9560\cdot51 \checkmark$$

8. An annuity involves saving €3000 per year at 7.3% p.a. for 8 years.

(i) Use the present value formula to calculate the single amount of money which could be invested at the same rate and for the same amount of time to give the same final amount.

(ii) Using the compound interest (future value) formula, find the final amount of the investment.

(iii) Using the future value annuity formula, check that the annuity gives the same final amount.

$P = \frac{F}{(1+i)^t}$

Series, $P =$

$S_n = \frac{a(1-r^n)}{1-r}$

1st €3000 is required now and wont be subject to interest

$\overset{T_8}{\frac{3000}{1.073^7}} + \dots + \overset{T_2}{\frac{3000}{1.073^1}} + \overset{T_1 = a}{3000}$

$a = 3000, \quad r = 1/1.073, \quad n = 8$

$S_8 = P = 3000 \frac{\left(1 - \left(\frac{1}{1.073}\right)^8\right)}{\left(1 - \frac{1}{1.073}\right)}$

$P = €19000.13$

8. An annuity involves saving €3000 per year at 7.3% p.a. for 8 years.

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(ii) Using the compound interest (future value) formula, find the final amount of the investment.

(iii) Using the future value annuity formula, check that the annuity gives the same final amount.

(ii)

$F = P(1+i)^t$

$S_n = \frac{a(1-r^n)}{1-r}$

What is the final amount worth?

$F = \overset{T_8}{3000(1.073)^8} + \dots + \overset{T_1 = a}{3000(1.073)^1}$

$a = 3219, \quad r = 1.073, \quad n = 8$

$S_8 = F = \frac{3219(1 - 1.073^8)}{1 - 1.073}$

Final amount = €33,385.23

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Check: does €19,000·13 amount to €33,385·23 for 8 years @ 7.3% p.a

$F = P(1+i)^t$

$F = (19000\cdot13)(1\cdot073)^8$

$= €33,385\cdot23$ ✓