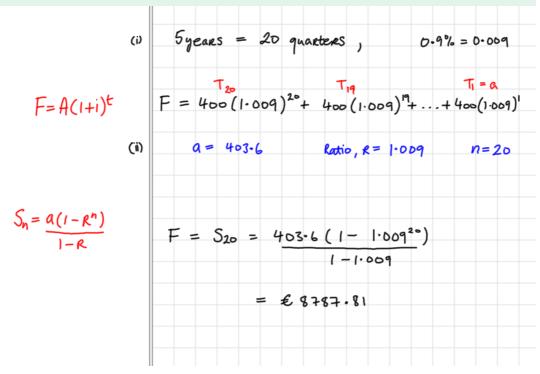
Financial Maths Section 5.3



Annuities (Instalment Saving)
& Pension Investments

Example 1

- (i) Represent her savings by a geometric series
- (ii) Find the value of her investment at the end of the period.



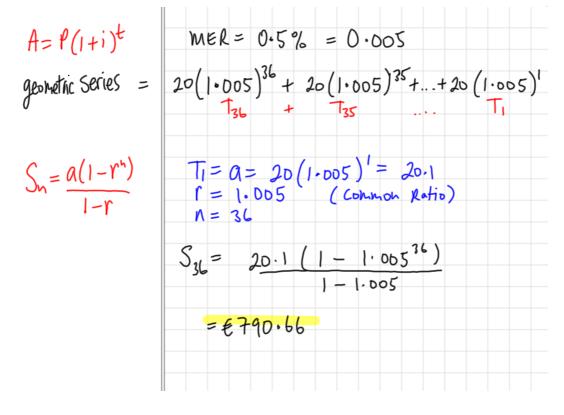
Example 2

Find the sum of money, €P, that needs to be saved per month to cover the cost of a €1500 holiday in 18 months time. The interest rate on offer is 0.4% per month.

The future value will be $\in 1500$ after 18 months of investing $\in P$ per month @ 0.4% M.E.R. $F = P(1+i)^{t}$ $I500 = P(1\cdot004)^{18} + P(1\cdot004)^{17} + ... + P(1\cdot004)^{1}$ $a = 1\cdot004P, Ratio R = 1\cdot004, N = 18$ $F = S_{18} = 1\cdot004P + P(1-1\cdot004)^{18} = 1500$ $1-1\cdot004$ $\Rightarrow 18\cdot6998P = 1500$ $P = 1500/18\cdot6998$ P = £80.21

Exercise 5.3

1. Calculate the future value of 36 monthly instalments of €20.00 at an interest rate of 0.5% per month. What is the total interest earned on these savings?



- Marie has saved €30.00 per month since her 18th birthday.
 If her bank has guaranteed her an interest rate of 4% per annum, find
 - (i) the equivalent monthly rate of interest, correct to two places of decimals
 - (ii) the value of her savings on her 21st birthday.

(i)	AER = 4% = 1 MER = ? = r
$(1+r)^{12} = (1+i)^{1}$	AER = 4% = 1 MER = ? = r $(1+r)^{12} = (1+1)^1 \Rightarrow r = \frac{12}{1+1} - 1$
	$meR = \frac{12\sqrt{ .04 }}{ .04 } = 0.00327$
time? terns? (ii)	18th . > 21st birthday = 3 years = 36 months
geometric Series=	$30(1.00327)^{36} + 30(1.00327)^{35} + + 30(1.00327)^{1}$ T_{36} $T_{1} = a$
	N=36, a=30(1.00327) =30.1, r=1.00327
$S_n = \underline{\alpha(1-r^n)}_{1-r}$	$S_{36} = \frac{30.1(1-1.00327^{36})}{1-1.00327} = £1148$

Example 3

What amount of money is needed now to provide a pension of €25 000 a year for 20 years, assuming an AER of 4%?

$$A = P(1+i)^{t}$$

$$\Rightarrow P = A$$

$$(1+i)^{t}$$

$$\text{Principals that each amount to} \approx 25000$$

$$\text{After amounts of time.}$$

$$Quantic Saies$$

$$Total = 25000 + 25000 + ... + 25000$$

$$(1.04)^{20} \quad (1.04)^{19} \quad T_{1}$$

$$T_{20} \quad T_{19} \quad T_{1}$$

$$A = T_{1}$$

$$A = 25000 / 1.04 = 24038.46$$

$$\Gamma = \frac{1}{1.04}$$

$$S_{10} = 24038.46 (1 - (1.04)^{20})$$

$$1 - (1.04)^{20}$$

$$1 - (1.04)^{20}$$

$$1 - (1.04)^{20}$$

$$1 - (1.04)^{20}$$

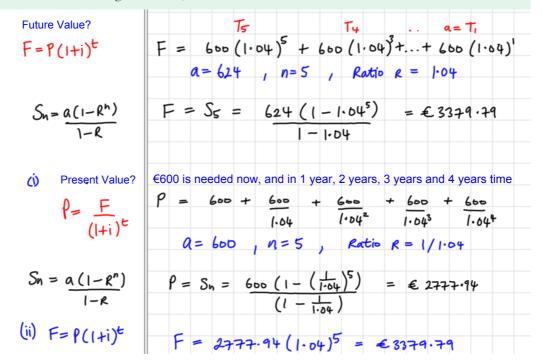
$$1 - (1.04)^{20}$$

$$1 - (1.04)^{20}$$

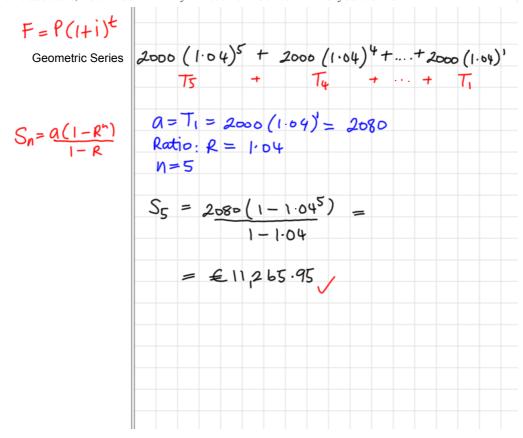
Example 4

Calculate the future value of an instalment savings plan based on saving €600 at the **start** of each year @ 4% per annum for 5 years.

- (i) Calculate the present value of these payments.
- (ii) Hence show that if the present value was put on deposit at the same rate for the same length of time, it would have the same future value.

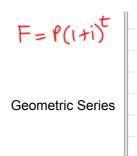


3. A special savings account offers an AER of 4% per annum. If I invest €2000 per year in this account, how much will my investment be worth in 5 years time?



Show that the future value of a series of n payments of $\in P$, earning an interest rate of i% per annum, can be written as:

Future value =
$$P(1 + i) \left(\frac{(1 + i)^n - 1}{i} \right)$$



$$S_n = \underbrace{\alpha(1-R^n)}_{1-R}$$

F=P(1+i)^t
assume n payments means ?
I per year for n years

$$P(1+i)^n + P(1+i)^{n-i} + ... + P(1+i)^i$$
 $T_n = T_{n-i}$

$$S_{n} = \underbrace{a(1-R^{n})}_{1-R}$$

$$R = T_{i} = P(1+i)$$

$$R = 1+i$$

$$n = n$$

$$S_{N} = \frac{P(1+i)(1-(1+i)^{h})}{1-(1-i)}$$

$$= P(1+i)(1-(1+i)^{h})$$

$$= P(1+i)(1-(1+i)^{h})$$

 $= \rho(1+i)\left(\frac{1+i}{n-1}\right)$

Show that the present value of a series of n payments of $\in \mathbb{P}$, earning an interest rate of i% per annum, can be written as:

Present value =
$$\frac{P}{(1+i)^n} \left(\frac{(1+i)^n - 1}{i} \right)$$

$$F = f(1+i)^n$$

$$P = \frac{F}{(1+i)^n}$$

Present Value =
$$S_n = \frac{a(1-R^n)}{1-R^n}$$

expand denominator

multiply above and below by (1+i)ⁿ

$$P = \frac{T_{n}}{F} + \frac{T_{n-1}}{F} + \dots + \frac{F}{F}$$

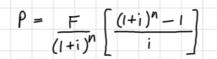
$$(1+i)^{n} \quad (1+i)^{n-1} \quad (1+i)^{1}$$

$$a = \frac{F}{(l+i)} \quad \text{Ractio} \quad R = \frac{1}{(l+i)} \quad (l+i)$$

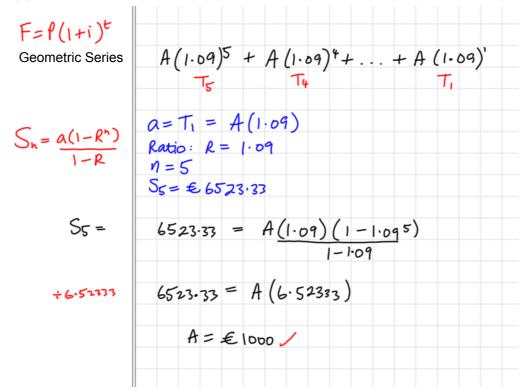
$$P = \frac{F}{1+i} \left(\frac{1 - \left[\frac{1}{(1+i)} \right]^n}{1 - \left(\frac{1}{1+i} \right)} \right)$$

$$P = F\left(\frac{1 - \frac{1}{(1+i)^n}}{1+i - 1}\right) = F\left(\frac{1 - \frac{1}{(1+i)^n}}{i}\right)$$

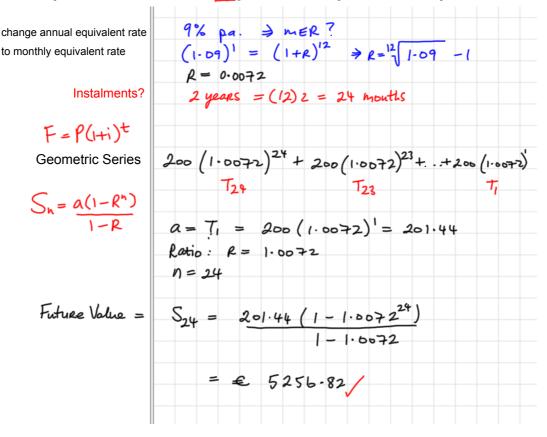
QED



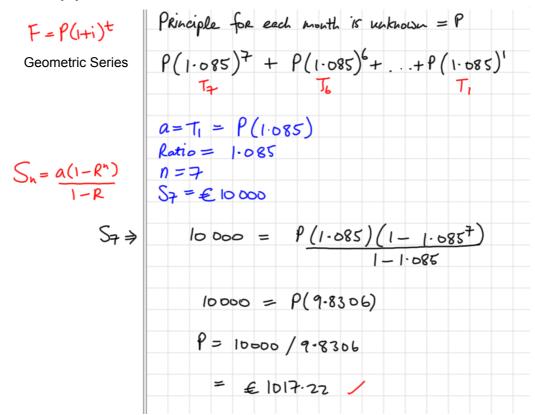
- 6. Anne received a cheque in the post for €6523.33 after saving for 5 years with her bank in a scheme offering 9% per annum. If she invested €A per annum,
 - (i) write down a geometric series representing the value of her investment over the 5 years
 - (ii) find the value of A.



7. Use the future value formula to find the final value if €200 is invested every month for 2 years. The interest rate is 9% per annum, compounded monthly.



8. George wants to make regular payments into an account that pays 8.5% per annum compound interest in order to have €10 000 after 7 years. Find the amount of each annual payment.



9. Ella wants to have €5000 in 3 years time. She invests in an annuity that pays 7.2% per annum, compounded quarterly. How much does she need to deposit each quarter to achieve her target of €5000?

$$(1+i)^{1} = (1+k)^{\frac{1}{4}} \qquad 7 \cdot 2^{\frac{1}{4}} \quad \text{p.a.} \Rightarrow \text{quarterly equivalent Rate? (R)}$$

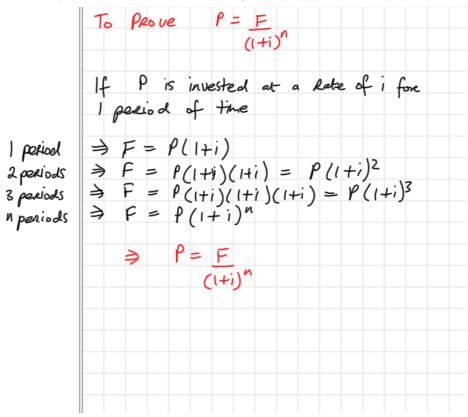
$$\text{number of instalments?}$$

$$\text{number of instalments?}$$

$$\text{3. years} = 3(4) = 12 \quad \text{quarters}$$

$$\text{P. (1.0175)}^{12} + P. (1.0175)^{11} + \dots + P. (1.0175)^{11$$

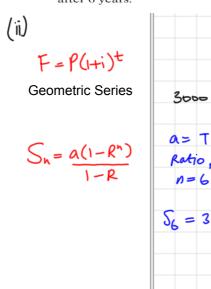
- 10. Prove that the present value of an annuity (instalments paid at the beginning of each period) is given by:
 - Future value (calculated at the end of each period) \div $(1 + i)^n$.

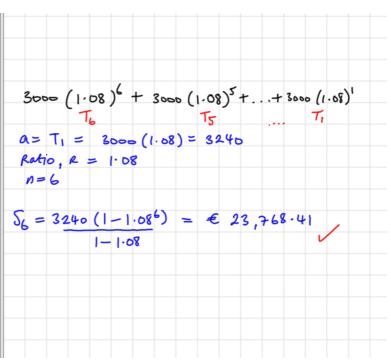


- 11. Show how the present value of an annuity involving depositing €3000 per year in an account for 6 years can be written as a geometric series, given that the interest rate is 8% per annum.
 - (i) Calculate the present value.
 - (ii) Calculate the future value of the annuity.
 - (iii) If the present value of the annuity in (i) was put on deposit as a single investment at 8% per annum, show that it will amount to the same future value of the annuity after 6 years.

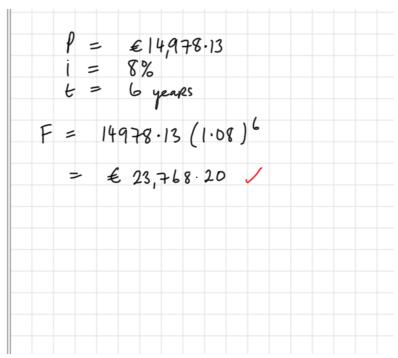
ώ	The first instalment is €3000 this is needed
ρ= <u>F</u> (1+i) ^t	immediately and won't be earning interest
Geometric Series	$\frac{3000}{(1.08)^5}$ + $\frac{3000}{(1.08)^4}$ + $\frac{3000}{(1.08)^4}$
$S_n = \underbrace{\alpha(1-R^n)}_{1-R}$	$T_1 = a = 3000$ $R = \frac{1}{1.08}$ $N = 6$
Present Value =	$S_6 = \frac{3000 \left(1 - \left(\frac{1}{1 \cdot 08}\right)^6\right)}{1 - \left(\frac{1}{1 \cdot 08}\right)}$
	= € 14,978.13

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 - (ii) Calculate the future value of the annuity.
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Summary

The future value of n payments of €P at i%

Future value = P(1 + i)
$$\left(\frac{1 - (1 + i)^n}{1 - (1 + i)}\right)$$

= P(1 + i) $\left(\frac{(1 + i)^n - 1}{i}\right)$

The present value (cost) of n payments of €P at i%

Present value
$$= \left(\frac{P}{1+i}\right) \left[\frac{1 - \left(\frac{1}{1+i}\right)^n}{1 - \left(\frac{1}{1+i}\right)}\right]$$
$$= \frac{P}{(1+i)^n} \left(\frac{(1+i)^n - 1}{i}\right)$$