# Theorems, Axioms and Corollaries

## Theorems

A **theorem** is a statement deduced from the axioms by logical argument.

- **Theorem 1** (Vertically-opposite Angles)
  Vertically opposite angles are equal in measure.
- **Theorem 2** (Isosceles Triangles)
  In an isosceles triangle the angles opposite the equal sides are equal.
  Conversely, if two angles are equal, then the triangle is isosceles.
- **Theorem 3** (Alternate Angles)
  If a transversal makes equal alternate angles on two lines, then the lines are parallel.
  Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.
- **Theorem 4** (Angle Sum 180)
  The angles in any triangle add to 180°.
- **Theorem 5** (Corresponding Angles)
  Two lines are parallel if and only if for any transversal, corresponding angles are equal.
- **Theorem 6** (Exterior Angle)
  Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
- **Theorem 7**
  The angle opposite the greater of two sides is greater than the angle opposite the lesser side.
  Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.
- **Theorem 8** (Triangle Inequality)
  Two sides of a triangle are together greater than the third.
- **Theorem 9**
  In a parallelogram, opposite sides are equal, and opposite angles are equal.
  Converse 1 to Theorem 9:
  If the opposite angles of a convex quadrilateral are equal, then it is a parallelogram.
  Converse 2 to Theorem 9:
  If the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.
- **Theorem 10**
  The diagonals of a parallelogram bisect one another.
- **Theorem 11**
  If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.
- **Theorem 12**
  Let ΔABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio.
- **Theorem 13**
  If two triangles ΔABC and ΔA' B' C' are similar, then their sides are proportional, in order.
- **Theorem 14** (Pythagoras)
  In a right-angle triangle the square of the hypotenuse is the sum of the squares of the other two sides.
- **Theorem 15** (Converse to Pythagoras)
  If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.
- **Theorem 16**
  For a triangle, base times height does not depend on the choice of base.
- **Theorem 17**
  A diagonal of a parallelogram bisects the area.
- **Theorem 18**
  The area of a parallelogram is the base by the height.
- **Theorem 19**
  The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.
- **Theorem 20**
  Each tangent is perpendicular to the radius that goes to the point of contact.
  If P lies on the circle, s, and a line l through P is perpendicular to the radius to P, then l is tangent to s.
- **Theorem 21**
  The perpendicular from the centre to a chord bisects the chord.
  The perpendicular bisector of a chord passes through the centre.

# Axioms

An **axiom** is a statement accepted without proof, as a basis for argument.

- **Axiom 1** (Two Points Axiom).
  There is exactly one line through any two given points.
- **Axiom 2** (Ruler Axiom).
  The distance between points has the following properties:
  the distance |AB| is never negative;
  |AB| = |BA|;
  if C lies on AB, between A and B, then |AB| = |AC| + |CB|;
  (marking off a distance) given any ray from A, and given any real number k ≥ 0, there is a unique point B on the ray whose distance from A is k.
- **Axiom 3** (Protractor Axiom).
  The number of degrees in an angle (also known as its degree-measure) is always a number between 0°and 360°. The number of degrees of an ordinary angle is less than 180°. It has these properties:
  A straight angle has 180°.
  Given a ray [AB, and a number d between 0 and 180, there is exactly one ray from A on each side of the line AB that makes an (ordinary) angle having d degrees with the ray [AB.
  If D is a point inside an angle ∠BAC, then |∠BAC|= |∠BAD| + |∠DAC|:
  Null angles are assigned 0°, full angles 360°, and reflex angles have more than 180°.
- **Axiom 4** (SAS+ASA+SSS).
  If (1) |AB|=|A'B'|, |AC|=|A'C'| and |∠A|=|∠A'|, or (2) |BC|=|B'C'|, |∠B|=|∠ B'|, and |∠ C| = |∠ C'|,
  Or (3) |AB| = |A'B'|, |BC| = |B'C'|, and |CA| = |C'A'|,
  then the triangles ΔABC and ΔA'B'C' are congruent.
- **Axiom 5** (Axiom of Parallels).
  Given any line l and a point P, there is exactly one line through P that is parallel to l.

# Corollaries

A **corollary** is a statement that follows readily from a previous theorem. Often a corollary is a statement of a theorem in a more specific context.

- **Corollary 1.** A diagonal divides a parallelogram into two congruent triangles
- **Corollary 2.** All angles at points of the circle, standing on the same arc, are equal.
- **Corollary 3.** Each angle in a semicircle is a right angle.
- **Corollary 4.** If the angle standing on a chord [BC] at some point of the circle is a right angle, then [BC] is a diameter.
- **Corollary 5.** If ABCD is a cyclic quadrilateral, then opposite angles sum to 180°.
  The converse also holds: If ABCD is a convex quadrilateral, and opposite angles sum to 180°, then it is cyclic.
- **Corollary 6** If two circles share a common tangent line at one point, then the two centres and that point are collinear.

A **proof** is a series of logical steps which we use to prove a theorem.

The **converse** of a theorem is the reverse of a theorem.
Example: In an isosceles triangles the angles opposite the equal sides are equal.
Converse: If two angles are equal in a triangle then the triangle is isosceles. Converse is true.

**Implies** is a term we use in a proof when we can write down a fact we have proved from our previuos statements. The symbol for implies is ⇒

**Congruent** triangles are triangles where all the corresponding sides and interior angles are equal in measure.

In **similar** or **equiangular** triangles all three angles in one triangle have the same measurement as the corresponding triangle.

**Is equivalent to** means something has the same value or measure as, or corresponds to, something elese. For example $3 is equivalent to €2.

**If and only if:** I will give you €100 if and only if you eat this apple. This means that if you eat this apple I'll give you €100 and if I have given you €100 you have eaten the apple.

**Proof by Contradiction** is where we cannot directly prove a statement but we can prove that the opposite statement is false.