LEAVING CERT Honours Maths notes on Algebra.

A polynomial expression is the addition or subtraction of many algebraic terms with positive integer powers.

The degree is the highest power of x. $3x^2 + 2x + 4$ has a degree of 2.

A linear expression has a degree of 1 $3x + 4$

A quadratic expression has a degree of 2 $x^2 + 4x + 2$

A cubic expression has a degree of 3 and so on. $X^3 + 2x^2 - 3x + 4$

The co-efficient is the number in front of a letter. $3x + 4 =$ the co-efficient of x is 3.

The number on its own is called the constant. $3x+4$ the constant is 4

When adding or subtracting algebraic terms only add like with like. + means I have and – means I owe.

When multiplying and dividing algebraic terms, If the signs are the same the answer is + and if the signs are different the answer will be negative. $2x(4) = 8x$ $-2(x) = -8x$ $-2(-x) = 2x$

Brackets mean multiply if there is no other instruction with the question.

$2x + (3x) = 5x$ (the instruction here is plus) $2x(3x) = 6x^2$ $x(x) = x^2$

When dividing algebraic terms always use the order of DIVIDE, MULTIPLY SUBTRACT, ADD.

Divide algebraic term as you would long division. Make sure the powers go down in order.

$$2x+5\sqrt{2x^3 + 3x^2 + 5x + 9}$$

There are six types of factorising:

1. Do they all have something in common? If so take out what they have in common, divide each number by this common factor and put your answer into brackets.
   $6a^2b - 9ab^2 = 3ab(2a - 3b)$

2. Is it the difference of two squares? (Two numbers, minus between them, everything squared) If so open up two brackets and add them in one bracket and subtract them in the other bracket.
   $25x^2 - 36y^2 = (5x+6y)(5x-6y)$
3. Are there four numbers that don't all have something in common? If so pair them off. Take out what each of them has in common and put the "leftovers " into brackets. The brackets MUST be identical.

$6pq + rs - 2sq - 3rp$		
$6pq - 2sq$	$rs - 3rp$	
$2q(3p - s)$	$r(s - 3p)$	(brackets not identical)
$2q(3p - s)$	$-r(-s+3p)$	(brackets identical)
$(3p - s)(2q - r)$		Answer.

4. Is it a quadratic equation? If so break up the first number, break up the last number and they must add or subtract to become the middle number. Watch the signs! $X^2 + 8x + 15 = (x + 5)(x + 3)$

5. Is it the difference of two cubes? $X^3 - y^3 = (x - y)(x^2 + xy + y^2)$

6. Is it the sum of two cubes? $X^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Adding, subtracting, multiplying and dividing fractions:

Adding and subtracting fractions always need a common denominator except when there is an equal sign.

- Always make sure your denominator is as simplified as possible before you begin to work with it

$$\frac{3}{x+5} - \frac{2}{x+3} + \frac{5x+19}{x^2+8x+15}$$

(x+3)(x+5)

$$\frac{3(x+3) - 2(x+5) + 1(5x+19)}{(x+5)(X+3)}$$

$$\frac{3x+9-2x-10+5x+19}{(x+5)(x+3)}$$

$$\frac{6x+18}{(x+5)(X+3)}$$

$$\frac{6(x+3)}{(x+5)(x+3)}$$

$$\frac{6}{x+5}$$

Multiplying fractions means multiplying top with top and bottom with bottom.

$$\frac{3}{x+5}\cdot\frac{x+3}{2}=\frac{3x+9}{2x+5}$$

NOTE: $$3\left(\frac{x+2}{x-4}\right) = \frac{3X+6}{X-4}$$

Never cancel a number out on a fraction unless it can cancel with every number

$\frac{3x+6}{9y-15} = \frac{x+2}{3y-5}$ you can cancel out here because ***every*** number can divide by 3.

Dividing fractions: Turn the second fraction upside down and multiply.

$$\frac{4x^2-10x}{9x^2+6x}\div\frac{2x-5}{3x+2}$$

$$\frac{4x^2-10x}{9x^2+6x}\cdot\frac{3x+2}{2x-5}$$

$$\frac{2x(2x-5)}{3x(3x+2)}\cdot\frac{3x+2}{2x-5}$$ (cancel out everything you can)

$$\frac{2}{3}$$

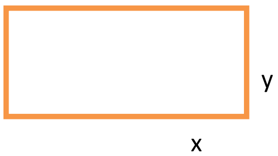
Dealing with hard fractions. $$\frac{1-\frac{3}{x}}{x-\frac{9}{x}} = 1 - \frac{3}{x} \div x - \frac{9}{x}$$

$$\frac{x-3}{x} \div \frac{x^2-9}{x} = \frac{x-3}{x}x\frac{x}{x^2-9}$$

$$\frac{x-3}{x} \times \frac{x}{(x+3)(x-3)} = \frac{1}{x+3}$$

Practical Application: In a rectangle the length is x and the perimeter is 8. Find the area in terms of x.

Answer:



Perimeter = 8

2x + 2y = 8

x + Y = 4

Y = 4 – X

Area = L x W

xy

x(4-x)

$4x - x^2$ Answer in terms of x as asked for.

<u>CHANGING THE SUBJECT OF THE FORMULA</u>

This involves rearranging the formula so that a particular letter is on its own.

Get rid of fractions (by multiplying everything by the common denominator)

Get rid of square roots by squaring both sides (once only on each side)

Example 1: $\frac{1}{b} + \frac{1}{a} = \frac{1}{c}$ Express c in terms of a and b.

multiply by abc (common denominator) to get ac + bc = ab

isolate c by factorising c(a+b) = ab

Get c on its own $c = \frac{ab}{a+b}$

Example 2 : $\sqrt[3]{\frac{3p-2}{2p+1}} = q$ Express p in terms of q

(cube both sides) $\frac{3p-2}{2p+1} = q^3$

Multiply both sides by 2p+1 $3p-2 = q^3(2p+1)$

Multiply out $3p-2 = 2pq^3 + q^3$

Bring anything with p to right hand side $3p - 2pq^3 = q^3 + 2$

Factorise to isolate p $p(3-2q^3) = q^3+2$

Get p on its own $p = \frac{q^3+2}{3-2q^3}$

SURDS

(i) $\sqrt{8 \pm 4} \neq \sqrt{8} \pm \sqrt{4}$ (ii) $\sqrt{32} = \sqrt{8} \times \sqrt{4}$

(iii) $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

To get rid of a square root underneath a fraction, multiply the top and the bottom by that square root.

NOTE: It is always better to have a complicated top number than a complicated bottom number in a fraction.

$\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} x \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

If you have a complicated denominator with a surd, we multiply on the top and the bottom by the complex conjugate which means we change the sign in the middle of the bottom number and multiply by that.

Example: $\frac{-1+\sqrt{3}}{1+\sqrt{3}}$

Change the sign in the middle on the bottom number 1-$\sqrt{3}$

Multiply top and bottom with this number $\frac{-1+\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$$\frac{-1+\sqrt{3}+\sqrt{3}-3}{1-\sqrt{3}+\sqrt{3}-3}$$

$$\frac{-4+2\sqrt{3}}{-2}$$

Change signs on the top and bottom and divide everything by 2 2 - $\sqrt{3}$

You can only add like surds.

$\sqrt{18} + \sqrt{50} - \sqrt{8} = \sqrt{2}\sqrt{9} + \sqrt{2}\sqrt{25} - \sqrt{4}\sqrt{2}$

$3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2}$ (now you can add them as they are all alike)

$6\sqrt{2}$

Surd equations.

Get rid of the square root by squaring both sides.

If there are two square roots in the question put one on either side and you will have to square across the equal sign twice

Example1:Solve $\sqrt{2x+1}$ = 3

2x+1 = 9

2x = 8

X = 4

Solve$\sqrt{5x+6}$ = $\sqrt{2x}$ +2

5x+6 = 2x +4$\sqrt{2x}$ + 4 (square across)

5x-2x + 6 – 4 = 4$\sqrt{2x}$

3x+2 = 4$\sqrt{2x}$ (square across)

$9x^2$ + 12x + 4 = 16(2x)

$9x^2$-20x + 4 = 0

-b Formula to get x = 2 and x = 0.22222

SIMULTANEOUS EQUATIONS.

There are three types of simultaneous equations. Simultaneous equations are used when you have two or three unknowns.

Type one: Each line has an "x", a "y" and a number. Put the two equations underneath each other making sure x is under x, y is under y and the numbers are also underneath each other.

Choose to eliminate x or y. Make the numbers the same by multiplying, make the signs different and add. You should now have found one letter and sub in this value to find the other letter.

Example: Solve for x and y 4x + 3y = -23 and x + 2y = -12.

Answer: 4x + 3y = -23

X + 2y = -12 Choose to work with x so multiply by 4 to make it the same as the top line.

4x + 3y = -23

4x + 8y = -48 Change the signs on the bottom line

4x + 3y = -23

-4x -8y = $^{+}$48 now add.

0x - 5y = 25 therefore 5y = -25 therefore y = -5

Now sub this value into the first equation to find x

4x + 3y = -23

4x -15 = -23

4x = -23 + 15

4x = -8 therefore answer x = -2.

Type two : This is when you have two equations , one linear and the other is not.

Step 1. Take the simple equation and get x/y on its own.

Step 2. Sub this value into the other equation, simplify and factorise until you have one of the values.

Step 3. Go back to step one and sub in the value you got in step two to find the other value.

Example: Solve for x and y the equations $x + 2y = 5$ and $x^2 + y^2 = 10$.

Answer: Step 1: $x + 2y = 5$ becomes $x = 5 - 2y$ (get x on its own)

Step 2. $x^2 + y^2 = 10$

$(5 – 2y)^2 + y^2 = 10$ Sub in value in step 1

$25 - 20y + 4y^2 + y^2 = 10$

$5y^2 - 20y + 25 - 10 = 0$

$5y^2 - 20y + 15 = 0$ Simplifying

$(5y - 5)(y - 3) = 0$ Factorising

$Y = 1$ and $y = 3$ Solving for y

Step 3: $x = 5 - 2y$ from step one

$X = 5 - 2(1)$ and $x = 5 - 2(3)$ subbing in values for y

$X = 3$ and $x = -1$ solving for x

Answers are (3,1) and (-1,3).

Type three:

This is when you are asked to solve three unknowns. You must be given three equations to do this. Take two pairs and eliminate the same variable in both. You are now doing an ordinary simultaneous equation as in type 1.

Solve $x + 2y - z = -1$ $2x + y + 3z = 14$ and $3x - y - z = -14$

Take two pairs and eliminate z in both.

$X + 2y – z = -1$ (x3)	$x + 2y - z = -1$
$\underline{2x + y + 3z = 14}$	$\underline{3x - y - z = -14}$
$3x + 6y - 3z = -3$	$x + 2y - z = -1$
$\underline{2x + y + 3z = 14}$	$\underline{-3x + y + z = 14}$
$5x + 7y = 11$	$-2x + 3y = 13$ This is now like Type one with 2

Solve by multiplying the first equation by 2 and the second equation by 5 to eliminate x and find y = 3 and then sub this value into either equation to find x = -2.

WORKING WITH ALGEBRAIC IDENTITIES.

When you have been given an equation with terms on both sides of the equal sign and you are asked to find the value of the letters then equate like with like.

Example one: Find the values of a and b given that $(2x+a)^2 = 4x^2 + 12x + b$.

$4x^2+4ax+a^2 = 4x^2 + 12x + b$ (equate x^2 with x^2 x with x number with number)

$4ax + 12x$ $a^2 = b$

$a = 3$ $3^2 = b$ (b = 9)

Example two:

If $3t^2x - 3px + c - 2t^3 = 0$ for all values of x, find c in terms of p

$3t^2x - 3px = 0x$ $c - 2t^3 = 0$

$t^2 - p = 0$ $c - 2tt^2 = 0$

$t^2 = p$ $c - 2tp = 0$

$t = \sqrt{p}$ $c = 2p\sqrt{p}$

Example three:

Given $\frac{1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$ for all values of x, find the values of A and B.

$$\frac{A}{(x+1)} + \frac{B}{(x-2)} = \frac{A(x-2)+B(x+1)}{(x+1)(x-2)}$$

$$\frac{1}{(x+1)(x-2)} = \frac{Ax-2A+Bx+B}{(+12)(x-2)}$$

$1 + 0x = Ax + Bx - 2A + B$

$1 = -2A + B$ $0x = Ax + Bx$

$1 = -2(-B) + B$ $0 = A + B \rightarrow A = -B$

$1 = 3B$

$\frac{1}{3} = B$ $A = \frac{-1}{3}$

Example four:

Given that $(x-t)^2$ is a factor of $x^3 + 3px + c$, show that $p = -t^2$ and $c = 2t^3$ (there must be three factors on each side – since there is only two on the left we must add one in to make the equation equal)

$(x-t)^2(x+a) = x^3 + 3px + c$

$(X^2 - 2xt + t^2)(x+a) = x^3 + 3px + c$

$X^3 + x^2a - 2x^2t - 2axt + t^2x + at^2 = x^3 + 3px + c$

$X^3 = x^3$	$x^2a - 2x^2t = 0x^2$	$t^2x - 2axt = 3px$	$at^2 = c$
	$a - 2t = 0$	$t^2 - 2at = 3p$	$at^2 = c$
	$a = 2t \rightarrow$	$t^2 - 2(2t) = 3p$	$(2t)t^2 = c$
		$t^2 - 4t^2 = 3p$	$\underline{2t^3 = c}$
		$-3t^2 = 3p$	
		$\underline{-t^2 = p}$	

SOLVING EQUATIONS:

Solve $2x + 4 = 6$ $\quad 2x = 6-4 \quad 2x = 2 \quad x = 1$

When you are asked to solve an equation which has an x^2 in it, try to factorise it first.

Example: Solve

$x^2 - 5x - 6 = 0$		$y^2 - 5y = 0$	
$(x-6)(x+1) = 0$		$y(y-5) = 0$	
$x-6 = 0$	$x+1 = 0$	$y = 0$	$y-5 = 0$
$x = 6$	$x = -1$	$y = 0$	$y = 5$

Let each bracket equal 0 separately and solve.

If you have a quadratic and you can't factorise it then use the "-b" formula on it to solve it.

Example : $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ this formula is on page 20 of the log tables.

REAL ROOTS OR IMAGINARY ROOTS

If you are asked to see are roots real, equal or imaginary, then find b^2-4ac

If your answer >0 (positive), the roots are real.

If the answer = 0, then the roots are equal and real

If the answer < 0, (negative), then the roots are imaginary.

FINDING THE EQUATION GIVEN THE ROOTS

If you are given the roots and asked to form the equation then the formula is

$x^2 - x\ (\text{sum}) + \text{product} = 0$

Example find the equation whose roots are 2 and -4

$X^2 - x(-2) + -8 = 0$ $\quad x^2 + 2x - 8 = 0$

CUBIC EQUATIONS:

If you are asked to find the factors or roots of a cubic equation, you find the first root by trial and error, change it into a factor and divide to find the other two factors and then the other two roots.

Example: Find the roots of $2x^3 + x^2 - 13x + 6 = 0$.

Try $x = 0$ Answer $6 = 0$ (not true)

Try $x = 1$ Answer $-4 = 0$ (not true)

Try $x = -1$ Answer $18 = 0$ (not true)

Try $x = 2$ Answer $0 = 0$ (true) therefore $x = 2$ is a root or $x-2$ is a factor.

Now divide $2x^3 + x^2 - 13x + 6$ by $x-2$ to get $2x^2 + 5x - 3$ as an answer and factorise this to get $(x+3)(2x-1)$ as the other two factors and then the roots are $x = -3$ and $x = ½$

When you are solving equations you are finding where these curves cross over the x axis when they are drawn on a graph.

FINDING MAXIMUM AND MINIMUM POINTS FROM A GRAPH

Write your cubic equation in the form of $a(x-p)^2 + q$. (you can do this by getting the perfect square) The minimum point will be (p,q)

If you write it like $q - a(x-p)^2$ then (p,q) will be the maximum point.

Example 1 : Find the minimum point of $x^2 + 4x + 1$

Find the perfect square $\quad x^2 + 4x + 4 - 4 + 1 \quad x^2 + 4x + 4 - 3 \quad (x+2)(x+2) - 3$

$(x+2)^2 - 3$ $\quad$ minimum point (-2,-3)

Example 2: Find the maximum point for $-x^2 - 2x + 3$

Re arrange and find the perfect square. $\quad 3 - (x^2 + 2x) \quad 3 + 1 - (x^2 + 2x + 1)$

$4 - (x+1)(x+1) \quad 4 - (x+1)^2$ $\quad$ Maximum point = (-1,4)