Inequalities

$>$	greater than
$\geqslant$	greater than or equal to
$<$	less than
$\leqslant$	less than or equal to

Example 1

Solve the inequality $3x + 7 \geqslant x + 2, x \in Z$, and plot the solution on a number line.

$-x, -7$

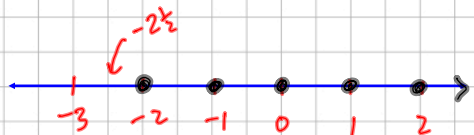
$3x + 7 \geq x + 2$

$\div 2$

$2x \geq -5$

$x \geq -5/2$ OR $-2\frac{1}{2}$

Z = integer = whole no.



Example 2

Solve the inequality $\frac{1}{6}(x-1) \geqslant \frac{1}{3}(x-4), x \in \mathrm{R}$.
Graph your solution on a number line.

×6

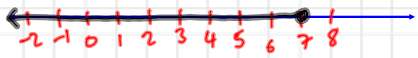
×−1

$\mathbb{R}$ = Real

$x - 1 \geq 2x - 8$ (−2x, +1 on both sides)

$-x \geq -7$

$x \leq 7$



Example 3

Solve the inequality $-9 < 3 - 4x \leqslant 1, x \in \mathrm{R}$.
Graph your solution on the number line.

−3

÷4

×−1

Rewrite

R = Real

$-9 < 3 - 4x \leq 1$ (−3 on all sides)

$\frac{-12}{4} < \frac{-4x}{4} \leq \frac{-2}{4}$

$-3 < -x \leq -\frac{1}{2}$

$3 > x \geq \frac{1}{2}$

$\frac{1}{2} \leq x < 3$

Example 4

(i) Find the solution set A, $\{x \mid 7 \leq 10 - 3x, x \in R\}$.
(ii) Find the solution set B, $\{x \mid 2 > \frac{4}{3} - 2x, x \in R\}$.
(iii) Find the set $A \cap B$ and graph the solution on the number line.

(i) A?

$+3x, -7$

$\div 3$

$7 \leq 10 - 3x$

$3x \leq 3$

$x \leq 1$

(ii) B?

$+2x, -2$

$\div 2$

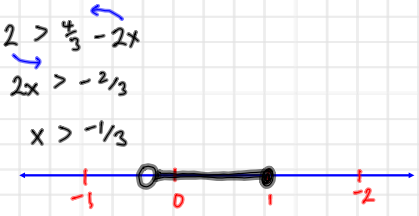
$2 > \frac{4}{3} - 2x$

$2x > -\frac{2}{3}$

$x > -\frac{1}{3}$

(iii) A∩B?

$-\frac{1}{3} < x \leq 1$



12. If $a < n < b$, and $100 < 2^n < 200$,
find the values of a and b, where $a, n, b \in N$.

$IN = \{1, 2, 3 \ldots\}$

$2^?$

$2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$

$2^6 = 64$, $2^7 = 128$, $2^8 = 256$

ANSWER: $6 < n < 8$

using logs

if $b^n = a$

$\Rightarrow n = \log_b a$

If $2^n = 100 \Rightarrow n = \log_2 100 = 6.64$

If $2^n = 200 \Rightarrow n = \log_2 200 = 7.64$

$6.64 < n < 7.64$

ANSWER: $6 < n < 8$

13. i Give one example to show that if $a > b > 0$ and $n > 0 \Rightarrow a^n > b^n$.
ii Now give an example to show that if $a > b > 0$ and $n < 0 \Rightarrow a^n < b^n$.
Write an equivalent set of conclusions for these:
iii If $a < b < 0$ and $n > 0 \ldots\ldots$,
iv but if $a < b < 0$ and $n < 0 \ldots\ldots$

(i) let $a = 2$, $b = 1$, $n = 1$

$a^n = 2^1 = 2$, $b^n = 1^1 = 1$ } Is $a^n > b^n$? $2 > 1$ yes ✓

(ii) let $a = 2$, $b = 1$, $n = -1$

$a^n = 2^{-1} = \frac{1}{2}$, $b^n = 1^{-1} = 1$ } Is $a^n < b^n$? $\frac{1}{2} < 1$ yes ✓

(iii) let $a = -2$, $b = -1$, $n = 1$

$a^n = (-2)^1 = -2$, $b^n = (-1)^1 = -1$ } Is $a^n < b^n$? $-2 < -1$ yes

Conclude $a^n < b^n$

(iv) let $a = -2$, $b = -1$, $n = -1$

$a^n = (-2)^{-1} = -\frac{1}{2}$, $b^n = (-1)^{-1} = -1$ } Is $a^n > b^n$? $-\frac{1}{2} > -1$ yes

Conclude $a^n > b^n$

Quadratic Inequalities

$>$	greater than
$\geqslant$	greater than or equal to
$<$	less than
$\leqslant$	less than or equal to

Example 1

Solve the inequality $x^2 - 2x - 8 \leq 0$.

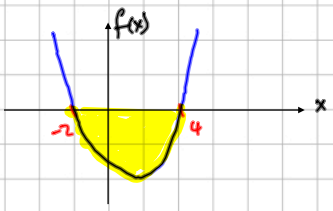
Solve if $=0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \quad , x = -2$

Sketch graph



Solution:

$-2 \leq x \leq 4$

algebra method
test $x=0$ value

$f(0) = (0)^2 - 2(0) - 8 = -8 \leq 0$ ✓ true

$\Rightarrow$ x is inside -2 and 4

$\Rightarrow -2 \leq x \leq 4$

Example 2

Find the range of values of k for which the equation $x^2 + (k-4)x + (k-1) = 0$ has real roots.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta \geq 0 \Rightarrow$ Real

Solve if $f(k) = 0$

Sketch

$a = 1$
$b = k-4$
$c = k-1$

$\Delta = (k-4)^2 - 4(1)(k-1) \geq 0$

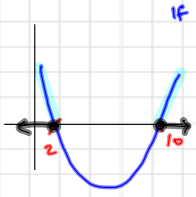
$k^2 - 8k + 16 - 4k + 4 \geq 0$

$k^2 - 12k + 20 \geq 0$

If $k^2 - 12k + 20 = 0$

$(k-10)(k-2) = 0$

$k = 10 \quad , k = 2$



$2 \geq k \geq 10$

Example 3

Find the range of values of x for which $\frac{2x+1}{x+2} < \frac{1}{2}$.

Is $(x+2)$ positive?
$(x+2)^2 \geq 0$

$2(x+2)^2\frac{(2x+1)}{(x+2)} < \frac{1}{2}(x+2)^2 2$

$(2x+4)(2x+1) < x^2+4x+4$

$4x^2+2x+8x+4 < x^2+4x+4$

$3x^2+6x < 0$

$x^2+2x < 0$

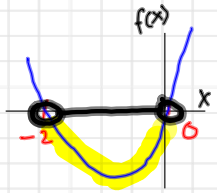
If $f(x)=0$?

$x^2+2x = 0$

$(x)(x+2)=0$

$x=0, \quad x=-2$

Sketch



ANSWER: $-2 < x < 0$