Leaving Cert Honours Maths

Integration

Practice Leaving Cert. Honours

Past Paper Questions on Integration

Lesson No. 1: Simple Algebraic Integration

2006

8 (a) Find (i) $\int \sqrt{x}dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

2005

8 (a) Find (i) $\int (2+x^3)dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

2004

8 (a) Find (i) $\int \frac{1}{x^2} dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

2003

8 (a) Find (i) $\int (x^3 + 2)\,dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

2002

8 (a) Find $\int (x^3 + \sqrt{x} + 2)\,dx$.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

2001

8 (a) Find (i) $\int \frac{1}{x^3} dx$

LESSON NO. 2: ALGEBRAIC INTEGRATION BY SUBSTITUTION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

2006

8 (b) Evaluate (i) $\int_1^2 x(1+x^2)^3 dx$

2005

8 (b) Evaluate (i) $\int_1^4 \frac{2x+1}{x^2+x+1}dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}dx = \ln|x| + C$$

2003

8 (b) (i) Evaluate $\int_0^1 \frac{2x}{\sqrt{1+x^2}}dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}dx = \ln|x| + C$$

2002

8 (b) Evaluate (i) $\int_2^7 \frac{2x-4}{x^2-4x+29}dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \frac{1}{x} dx = \ln|x| + C$

2001

8 (b) Evaluate (ii) $\int_0^4 \frac{(x+4)}{\sqrt{x^2+8x+1}}dx.$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \frac{1}{x} dx = \ln|x| + C$

Lesson No. 3: Exponential Integration

2006

8 (a) Find (ii) $\int e^{-2x} dx.$

$$\int e^x \, dx = e^x + C$$

2005

8 (a) Find (ii) $\int e^{3x} dx$

$$\int e^x \, dx = e^x + C$$

2003

8 (a) Find (ii) $\int e^{7x}\,dx.$

$$\int e^x\,dx = e^x + C$$

Lesson No. 4: Trigonometric Integration I

2004

8 (a) Find (ii) $\int \cos 6x\,dx$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

2001

8 (a) Find (ii) $\int \sin 5x\, dx$.

$\int \sin x\, dx = -\cos x + C$

$\int \cos x\, dx = \sin x + C$

2003

8 (c) (i) Show that $\int_{a}^{2a} \sin 2x\, dx = \sin 3a \sin a$.

$\int \sin x\, dx = -\cos x + C$

$\int \cos x\, dx = \sin x + C$

Sums $\rightarrow$ Products
$\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$
$\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

2006

8 (b) Evaluate (ii) $\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta \, d\theta$.

Sums $\rightarrow$ Products
$\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$
$\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

Lesson No. 5: Trigonometric Integration II

2005

8 (b) Evaluate (ii) $\int_0^{\frac{\pi}{8}} \sin^2 2\theta \, d\theta$

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

2003

EVEN AND ODD POWER

8 (b) (ii) By letting $u = \sin x$, evaluate $\int_0^{\frac{\pi}{2}} \cos x \sin^6 x \, dx$.

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

2004

TWO ODD POWERS

8 (b) Evaluate (ii) $\int_0^{\frac{\pi}{3}} \sin x \cos^3 x \, dx$

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

Lesson No. 6: Specials

2001

8 (b) Evaluate (i) $\int_0^3 \frac{12}{x^2+9}\,dx$

$\int \frac{dx}{(a)^2+(x)^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

2002

8 (b) Evaluate (ii) $\int_2^7 \frac{1}{x^2-4x+29}\,dx.$

$\int \frac{dx}{(a)^2+(x\pm b)^2} = \frac{1}{a}\tan^{-1}\left(\frac{x\pm b}{a}\right)+c$

2004

8 (b) Evaluate (i) $\int_3^6 \frac{dx}{\sqrt{36-x^2}}$

$$\int \frac{dx}{\sqrt{(a)^2-(x)^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$$

2005

8 (c) (i) Evaluate $\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx.$

$$\int \frac{dx}{\sqrt{(a)^2-(x\pm b)^2}} = \sin^{-1}\left(\frac{x\pm b}{a}\right)+c$$

2006

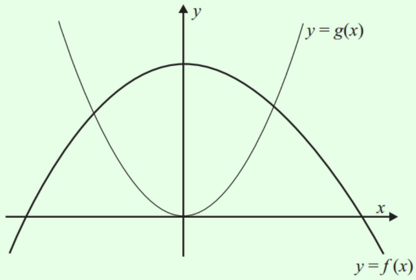
8 (c)

Lesson No. 7: Applications of Integration I: Area

The diagram shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = 12 - 3x^2$ and $g(x) = 9x^2$.

(i) Calculate the area of the region enclosed by the curve $y = f(x)$ and the x-axis.

(ii) Show that the region enclosed by the curves $y = f(x)$ and $y = g(x)$ has half that area.



$$A = \int_a^b y \, dx$$

2004

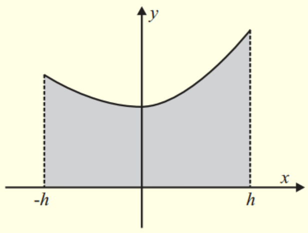
8 (c)

The graph of the function $f(x) = ax^2 + bx + c$ from $x = -h$ to $x = h$ is shown in the diagram.

(i) Show that the area of the shaded region is

$\frac{h}{3}[2ah^2 + 6c]$.

(ii) Given that $f(-h) = y_1$, $f(0) = y_2$ and $f(h) = y_3$, express the area of the shaded region in terms of y_1, y_2, y_3 and h.



$$A = \int_a^b y \, dx$$

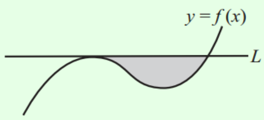
2002

8 (c)

Let $f(x) = x^3 - 3x^2 + 5$.

L is the tangent to the curve $y = f(x)$ at its local maximum point.

Find the area enclosed between L and the curve.



$$A = \int_a^b y\,dx$$

2001

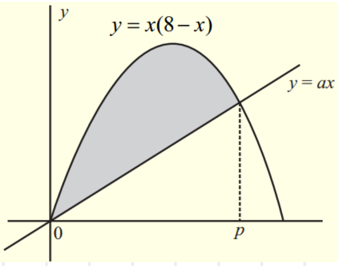
8 (c)

a is a real number such that $0 < a < 8$.

The line $y = ax$ intersects the curve $y = x(8 - x)$ at $x = 0$ and at $x = p$.

(i) Show that $p = 8 - a$.

(ii) Show that the area between the curve and the line is $\frac{p^3}{6}$ square units.



$$A = \int_a^b y\,dx$$

Lesson No. 8: Applications of Integration II: Volume

2005

8 (c) (ii) Use integration methods to derive a formula for the volume of a cone.

Revolution around the x-axis

$$V = \pi \int_a^b y^2 dx$$

2003

8 (c) (ii) Use integration methods to show that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

Revolution around the x-axis

$$V = \pi \int_a^b y^2 dx$$