LCHL 2012

Sum

8. **(a)** Find $\int (1+\cos 2x+e^{3x})\,dx.$

integrate

$$= x - \tfrac{1}{2}\sin 2x + \tfrac{1}{3}e^{3x} + c$$

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SUBSTITUTION

(b) **(i)** Evaluate $\int_1^3 \frac{12}{3x-2}\,dx.$

$u=?$ let $u = 3x-2$

$du=?$ $\frac{du}{dx} = 3$

$du = 3dx$

$\frac{1}{3}du = dx$

change limits

$u = 3x-2$

$u_1 = 3(3)-2 = 7$

$u_2 = 3(1)-2 = 1$

Rewrite $\Rightarrow$ Integral $= \int_1^7 \frac{12(\frac{1}{3})du}{u}$ $\quad 4\int_1^7 \frac{1}{u}du$

move constant outside of the integral

integrate $4\left[\ln u\right]_1^7 = 4[\ln 7 - \ln 1] = 4\ln\frac{7}{1}$

$= 4\ln 7$

Rewrite using

$\int (\sin^2 nx)\,dx$

$= \frac{1}{2}(1-\cos 2nx)\,dx$

Integrate

$\int \cos nx\,dx$

$= \frac{1}{n}\sin nx$

evaluate with constants

(ii) Evaluate $\int_0^{\frac{\pi}{8}} \sin^2 2x\,dx.$

$= \frac{1}{2}\int_0^{\frac{\pi}{8}} (1-\cos 4x)\,dx$

$= \frac{1}{2}\left[x + \frac{1}{4}\sin 4x\right]_0^{\pi/8}$

$= \frac{1}{2}\left[\left(\frac{\pi}{8} + \frac{1}{4}\sin 4\left(\frac{\pi}{8}\right)\right) - \left(0 + \frac{1}{4}\sin 4(0)\right)\right]$

$= \frac{1}{2}\left[\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0+0)\right] = \frac{\pi}{16} - \frac{1}{8}$ ✓