Polar Form of a Complex Number

Rectangular form / Cartesian form: $x + iy$
Polar form / Modulus argument form: $r(\cos\theta + i\sin\theta)$

15 Jan. 2013

r = modulus

θ = argument

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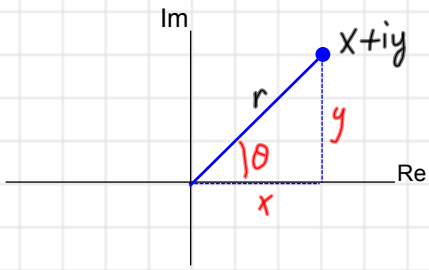


CAH $\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

SOH $\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

$\Rightarrow iy = ir\sin\theta$

$x + iy = r(\cos\theta + i\sin\theta)$

$x + iy$
$r(\cos\theta + i\sin\theta)$

Use calculator

[Radians]

Radians Degrees
$\frac{\pi}{6} \Rightarrow \frac{180}{6} = 30°$

Example 1

Express in the form $x + iy$ these complex numbers:
(a) $z_1 = 12\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ (b) $z_2 = 5\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

(a) $z_1 = 12\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 6\sqrt{3} + 6i$

(b) $z_2 = 5(0.924 + i\,0.383)$
$= 4.62 + 1.91\,i$

$x + iy$
$r(\cos\theta + i\sin\theta)$

r = modulus

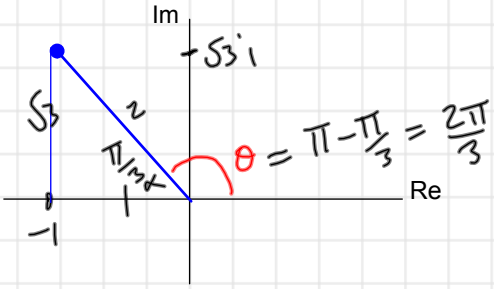
$|a+bi| = \sqrt{a^2+b^2}$

θ = argument

Example 2

Express $(-1 + i\sqrt{3})$ in the form $r(\cos\theta + i\sin\theta)$.

$r = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$



Polar Form $(-1 + i\sqrt{3}) = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

16-Jan-2013

Homework

Write the following in Polar Form

(1) $2 + 2i$

(2) $-3i$

(3) $-2 + i\sqrt{2}$

(4) $\frac{1}{2} - \frac{\sqrt{3}}{2} i$

(5) $(1 + i\sqrt{3})^2$

16-Jan-2013

Homework

Write the following in Polar Form

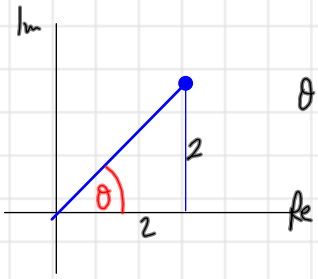
(1) $2 + 2i$

$r = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

r = modulus

$|a + bi| = \sqrt{a^2 + b^2}$

θ = argument



$\theta = 45^\circ = \frac{\pi}{4}$

$x + iy$

$r(\cos\theta + i\sin\theta)$

Polar Form

$2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Homework

r = modulus

$|a+bi| = \sqrt{a^2+b^2}$

θ = argument

$x + iy$

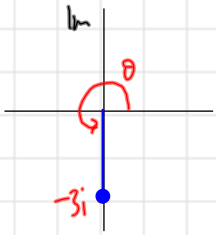
$r(\cos\theta + i\sin\theta)$

Polar Form

Write the following in Polar Form

(2) $-3i$

$r = \sqrt{3^2} = 3$



$\theta = \frac{3\pi}{2}$

$-3i = 3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

Homework

r = modulus

$|a+bi| = \sqrt{a^2+b^2}$

θ = argument

$x + iy$

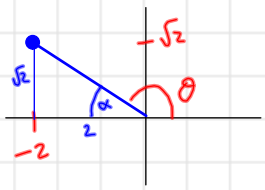
$r(\cos\theta + i\sin\theta)$

Polar Form

Write the following in Polar Form

(3) $-2 + i\sqrt{2}$

$r = \sqrt{2^2+\sqrt{2}^2} = \sqrt{4+2} = \sqrt{6}$



$\tan\alpha = \sqrt{2}/2$

$\alpha = \tan^{-1}(\sqrt{2}/2)$

$\alpha \approx 0.615$

$\theta = \pi - 0.615 = 2.53$

angle is usually nicer!

$-2+i\sqrt{2} = \sqrt{6}(\cos 2.53 + i\sin 2.53)$

Homework

r = modulus

$|a+bi| = \sqrt{a^2+b^2}$

θ = argument

$x + iy$

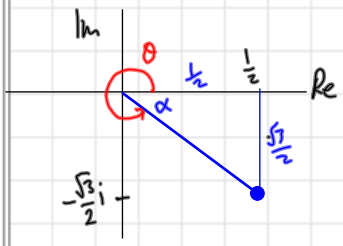
$r(\cos\theta + i\sin\theta)$

Polar Form

Write the following in Polar Form

④ $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1$



$\tan\alpha = \left(\frac{\sqrt{3}/2}{1/2}\right) = \sqrt{3}$

$\alpha = \tan^{-1}\sqrt{3}$

$= \pi/3$

$\theta = 2\pi - \pi/3 = 5\pi/3$

$\frac{1}{2} - \frac{\sqrt{3}}{2}i = 1\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

Homework

expand

$(a+b)^2 = a^2 + 2ab + b^2$

r = modulus

$|a+bi| = \sqrt{a^2+b^2}$

θ = argument

$x + iy$

$r(\cos\theta + i\sin\theta)$

Polar Form

Write the following in Polar Form

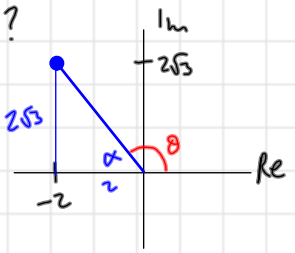
⑤ $(1+i\sqrt{3})^2$

$= 1 + 2\sqrt{3}i \mp 3i^2$

$= -2 + 2\sqrt{3}$

$r = \sqrt{2^2+(2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

θ = ?



$\alpha = \tan\left(\frac{2\sqrt{3}}{2}\right) = \tan\sqrt{3}$

$\alpha = \pi/3$

$\theta = \pi - \pi/3 = 2\pi/3$

$(1+i\sqrt{3})^2 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$