Probability

Fundamental Principal of Counting states that if one event has **m** possible outcomes and a second event has **n** possible outcomes then the total possible number of outcomes is: $\boldsymbol{m \times n}$

Arrangements

We often multiply a list of numbers in descending order using the box method. We must be careful when conditions are applied.
Example would be ways of rearranging the letters in the word BARTER
$[6] \times [5] \times [4] \times [3] \times [2] \times [1] = 720$
This can be done on the calculator using 6!

How many of these begin with a vowel?
$[2] \times [5] \times [4] \times [3] \times [2] \times [1] = 240$

How many don't begin with a vowel?
$720 - 240 = 480$

$$\binom{n}{r} = \binom{n}{n-r}$$

Combinations (Choosing Questions)

Ways of selecting items when the order is not important. The calculator has a button for this. Again we must be careful when conditions are applied.
Example would be selecting 3 people from a group of 8.

$$\binom{8}{3} = 8\,{}^{n}C_{r}\,3 = 56$$

Remember in maths the word AND generally means to multiply, while the word OR generally means to add.

How can we select a committee of 3 people from 5 boys and 4 girls if there must be at least one boy and one girl on the committee.

(2 Boys **And** 1 Girl) **OR** (1 Boy **AND** 2 Girls)

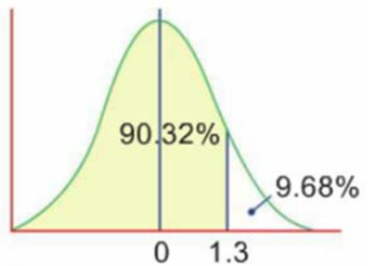
$$\left(\binom{5}{2} \times \binom{4}{1}\right) + \left(\binom{5}{1} \times \binom{4}{2}\right)$$

Standard Normal Tables

If the variables have a normal distribution we can use the tables to convert z-scores to probabilities

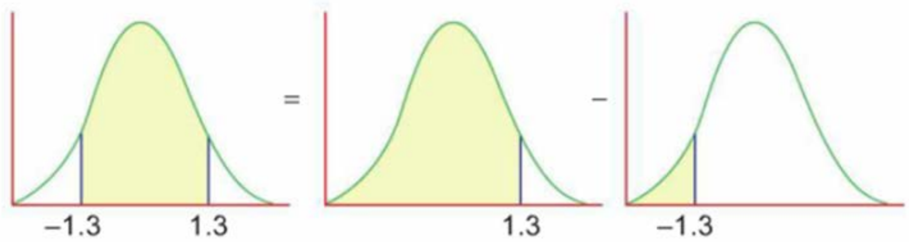
If we are asked Find $P \leq 1.3$ we are being asked to find the probability that the result will be less than or equal to 1.3 standard deviations from the mean. Checking the tables tells us
$P(z \leq 1.3) = 0.9032$

This means 90.32% of the population will give readings less than 1.3 standard deviations from the mean.



$$z = \frac{x - \mu}{\sigma}$$

If asked what the probability is that the result is **between** 1.3 standard deviations
Curve is symmetrical so we can use this to work out negative values of z.



$P(-1.3 \leq z \leq 1.3 = 0.9032 - 0.0968$
$= 0.8064$

A **trial** is the act of doing an experiment in probability.
An **outcome** is one of the possible results of the trial.
If all outcomes of a trial are equally likely then the trial is **fair** or **unbiased.** Example would be flipping a coin.
A **sample space** is a list all the possible outcomes (list, two way table, tree diagram)
The **event** is the occurrence of one or more specific outcomes.
The best way to deal with more than one event (combined events) is a diagram. (**Two Way Table** or **Tree Diagram**)

Dependent events are events where the 2nd event is affected by the 1st.
Independent events are where the outcome of the 1st does NOT affect the outcome of the second.

$$\boldsymbol{Relative\ Frequency} = \frac{number\ of\ times\ the\ event\ happens\ in\ a\ trial}{total\ number\ of\ trials}$$

$\boldsymbol{Expected\ Frequency}$ = number of trials × relative frequency or probability

$$\boldsymbol{Probability}\ of\ an\ event\ \boldsymbol{P(E)} = \frac{number\ of\ desirable\ outcomes}{total\ number\ of\ possible\ outcomes}$$

This is most often best left as a fraction, one number over another.

Mutually Exclusive have no outcomes in common.
Events that CANNOT occur at the same time.
Two events E and F are mutually exclusive if $E \cap F = \emptyset$

In **OR** events we ADD the probabilities (events are NOT mutually exclusive)

$$P(E \cup F) = P(E\ or\ F) = P(E) + P(F) - P(E \cap F)$$

In **AND** events we MULTIPLY the probabilities (events are mutually exclusive)

Conditional Probability

$$P(A|B) = \frac{P(E \cap F)}{P(B)}$$

Probability of A given that we know it is B is the probability that it is A and B over the Probability that it is B

Expected Value E(X) is the average outcome of an event

$$E(X) = \sum x.P(x)$$

To find we multiply every possible outcome by the probability for that outcome and then add all these values together.

The expected value on rolling a die is

$$1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) = 3\frac{1}{2}$$

Bernoulli Trial
There are two outcomes: success or failure
The trials are independent.
The probability of success does not change from one trial to another.
Example would be scoring or missing a penalty.

$$\binom{n}{r} p^r q^{n-r}$$

p = probability of success
q = probability of failure
n = number of trials
r = number of desired outcomes