The Circle

Class Notes

Jan 2012

Q6 Given: $r = \sqrt{10}$, contains $(1,2)$ and $(-1,4)$

P.248 Find 2 circles

$\sqrt{10} = \sqrt{f^2+g^2-c}$

① $10 = f^2+g^2-c$

$(1)^2+(2)^2+2g(1)+2f(2)+c=0$

② $2g+4f+c=-5$

$(-1)^2+(4)^2+2g(-1)+2f(4)+c=0$

③ $-2g+8f+c=-17$

$f^2+g^2-\cancel{c}=+10$

$4f+2g+\cancel{c}=-5$

④ $f^2+4f+g^2+2g=5$

$2g+4f+c=-5$

$2g-8f-c=17$

$4g-4f=12$

$g-f=3$

⑤ $g=3+f$

$f^2+4f+(3+f)^2+2(3+f)=5$

$f^2+4f+9+f^2+6f+6+2f=5$

$2f^2+12f+10=0$

$f^2+6f+5=0$

$(f+1)(f+5)=0$

$f=-1, -5$

$\Rightarrow g=2, -2$

$c=-2g-4f-5$

$\Rightarrow c=-2(2)-4(-1)-5=-5$

or $c=-2(-2)-4(-5)-5=+19$

$x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+4x-2y-5=0$

$x^2+y^2-4x-10y+19=0$

2005

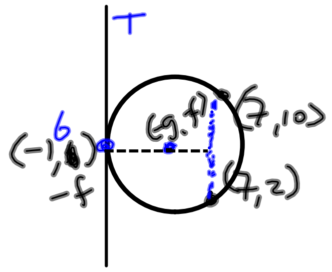
1 (c) A circle passes through the points (7, 2) and (7, 10). The line $x = -1$ is a tangent to the circle. Find the equation of the circle.

$7^2+2^2+7g2+2f2+c=0$

$14g+4f+c=-53$

$7^2+10^2+7g2+10f2+c=0$

$14g+20f+c=-149$



$\sqrt{(-g+1)^2+(0)^2}=\sqrt{g^2+f^2-c}$

$\cancel{g^2}-2g+1^2=\cancel{g^2}+f^2-c$

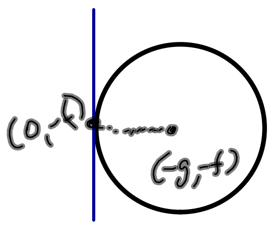
$f^2+2g-c=+1$

2004

1 (c) The y-axis is a tangent to the circle $x^2+y^2+2gx+2fy+c=0$.

(i) Prove that $f^2=c$.

(ii) Find the equations of the circles that pass through the points $(-3, 6)$ and $(-6, 3)$ and have the y-axis as a tangent.



$r=\sqrt{(-g-0)^2+(0)^2}=\sqrt{g^2}$

$r=\sqrt{g^2+f^2-c}=\sqrt{g^2}$

$\Rightarrow \cancel{g^2}+f^2-c=\cancel{g^2}$

1 $f^2=c$

$(-3)^2+6^2+2g(-3)+2f(6)+c=0$

2 $-6g+12f+c=-45$

$(-6)^2+(3)^2+2g(-6)+2f(3)+c=0$

3 $-12g+6f+c=-45$

$\cancel{12g}-24f-2c=90$

$-\cancel{12g}+6f+c=-45$

$-18f-c=45$

4 $18f+c=-45$

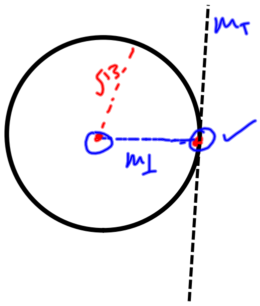
$18f+f^2+45=0$

$f=-3,-15$

Find equation of tangent?

Given: Equation of circle and tangent point

Q7 p.250



$r = \sqrt{3^2+1^2+3} = \sqrt{13}$

$d = \sqrt{(2)^2+(3)^2} = \sqrt{13}$

$m_\perp = \frac{3}{2} \perp -\frac{2}{3} = m_T$

$y - y_1 = m(x - x_1)$

$y + 4 = -\frac{2}{3}(x+5)$

$3y + 12 = -2x - 10$

$2x + 3y + 22 = 0$

$x^2+y^2+6x+2y-3=0$

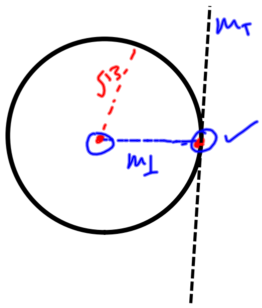
P.$(-5,-4)$

centre $(-3,-1)$

Find equation of tangent?

Given: Equation of circle and tangent point

Q6 p.250



$r = \sqrt{\quad} = \sqrt{29}$

$d = \sqrt{(\quad)^2+(\quad)^2} = 29$

+-

$y - y_1 = m(x - x_1)$

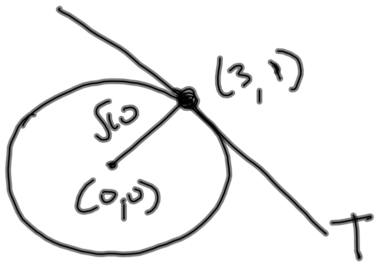
P.(   )

centre (   )

$x^2 + y^2 = 10$

$(3, 1)$

$\boxed{P.250 \quad Q1}$



$m_\perp = \frac{1}{3} \perp = -3 = m_T$

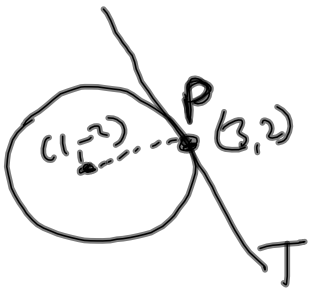
$$y - y_1 = m(x - x_1)$$
$$y - 1 = -3(x - 3)$$
$$y - 1 = -3x + 9$$
$$3x + y - 10 = 0$$

Q8

$x^2 + y^2 - 2x + 4y - 15 = 0$

? $P(3, 2)$

$c = (1, -2) \qquad R = \sqrt{1^2 + 2^2 + 15} = \sqrt{20} = 2\sqrt{5}$



$m_\perp = \frac{4}{2} = 2 \perp -\frac{1}{2} = m_T$

$$y - y = m(x - x_1)$$
$$y - 2 = -\tfrac{1}{2}(x - 3)$$
$$2y - 4 = -x + 3$$
$$x + 2y - 7 = 0$$ ✓

P.19 given $x^2+y^2+2gx+2fy+c=0$

centre = $(-g,-f)$

Radius = $\sqrt{g^2+f^2-c}$

tangent at (x_1,y_1)

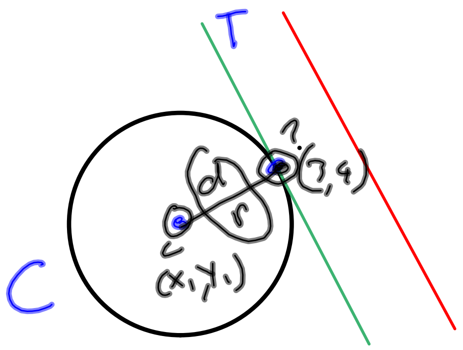
$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

note if centre $(0,0)$ $xx_1+yy_1+c=0$

$$x^2+y^2-2x+4y-15=0$$
pt $(3,2)$

Centre $(1,-2)$ $g=-1,\ f=2$
$x_1=3,\ y_1=2$

$$3x+2y+-1(x+3)+2(y+2)-15=0$$
$$3x+2y-x-3+2y+4-15=0$$
$$2x+4y-14=0$$
$$x+2y-7=0$$



Given : Circle $x^2+y^2=25$
: Line $3x+4y-25=0$

Is Line Tangent?

distance from Line to centre $= \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \dfrac{|3(0)+4(0)-25|}{\sqrt{3^2+4^2}}$

$$= \frac{25}{\sqrt{25}} = \frac{25}{5} = 5$$

Radius = 5 $\Rightarrow$ Tangent