Simplify each of the following algebraic expressions.

(i) $\dfrac{12m^2n^3}{(6m^4n^5)^2}$

$= \dfrac{12m^2n^3}{36m^8n^{10}}$

$= \dfrac{1}{4m^6n^7}$

(ii) $\dfrac{3+\frac{1}{x}}{\frac{5}{x}+4}$

$= \dfrac{3x+1}{5+4x}$

Divide $3x^4 - 9x^2 + 27x - 66$ by $x - 2$.

$$
\begin{array}{r}
3x^3+6x^2+3x+33 \\
x-2\,\overline{)\,3x^4+0x^3-9x^2+27x-66} \\
\mp 3x^4 \pm 6x^3 \\
\hline
6x^3-9x^2 \\
\mp 6x^3 \pm 12x^2 \\
\hline
3x^2+27x \\
\mp 3x^2 \pm 6x \\
\hline
33x-66 \\
\mp 33x \pm 66 \\
\hline
\end{array}
$$

Solve the simultaneous equations ① $3x + 5y - z = -3$
② $2x + y - 3z = -9$
③ $x + 3y + 2z = 7$.

2① + ③ $6x + 10y - 2z = -6$
$x + 3y + 2z = 7$
④ $7x + 13y = 1$

3① − ② $9x + 15y - 3z = -9$
$-2x - y + 3z = 9$
$7x + 14y = 0$
⑤ $x + 2y = 0$

④ − 7⑤ $7x + 13y = 1$
$-7x - 14y = 0$
$-y = 1$
$y = -1$

$x + 2(-1) = 0$
$x = 2$

$3(2) + 5(-1) - z = -3$
$6 - 5 - z = -3$
$1 - z = -3$
$z = 4$

Factorise $x^3 - 27$.

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$x^3 - 27 = (x-3)(x^2 + 3x + 9)$

Simplify $(b+1)^3 - (b-1)^3$.

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$(b+1)^3 - (b-1)^3$

$= \big((b+1)-(b-1)\big)\big((b+1)^2 + (b+1)(b-1) + (b-1)^2\big)$

$= (b+1-b+1)\big((b^2+2b+1) + (b^2-1) + (b^2-2b+1)\big)$

$= (2)(3b^2+1)$

Find the rule for each of the following quadratic patterns.

(i) 3, 12, 27, 48, 75 ...

(ii) 5, 20, 45, 80, 125 ...

(i)

x	0	1	2	3	4
f(x)	3 = c	12	27	48	75
1st Difference		9	15	21	27
2nd Difference			2a = 6	6	6

QUADRATIC

$a = \frac{6}{3} = 3$

$b = 6$

$c = 3$

$f(x) = ax^2 + bx + c$

$f(1) = 12 \Rightarrow 3(1)^2 + b(1) + 3 = 12$

$b + 6 = 12 \Rightarrow b = 6$

Rule: $f(x) = 3x^2 + 6x + 3$

(ii)

x	0	1	2	3	4
f(x)	5 = c	20	45	80	125
1st Difference		15	25	35	45
2nd Difference			2a = 10	10	10

QUADRATIC

$a = \frac{10}{2} = 5$

$b = 10$

$c = 5$

$f(x) = ax^2 + bx + c$

$f(1) = 20 \Rightarrow 5(1)^2 + b(1) + 5 = 20$

$b + 10 = 20 \Rightarrow b = 10$

$f(x) = 5x^2 + 10x + 5$

If, for all values of x, $(3p - 2t)x + r - 4t^2 = 0$, show that $r = 9p^2$.

$(3p - 2t)x + (r - 4t^2) = 0x + 0$

$\Rightarrow 3p - 2t = 0$

$3p = 2t$

$\frac{3p}{2} = t$ (1)

$\Rightarrow r - 4t^2 = 0$

$r = 4t^2$ (2)

(1) (2) $r = 4\left(\frac{3p}{2}\right)^2 = 4\left(\frac{9p^2}{4}\right) = 9p^2$

In a chemistry class, a group of students need a 15% acid solution to complete a test. The lab only has 10% acid solution and 30% acid solution. The students decide to mix the two solutions to get the 15% solution they require.
If the students need 10 litres of the new solution, find

(i) the number of litres of the 10% solution they require
(ii) the number of litres of the 30% solution they require.

let x = Volume of 10% Solution (litres)
y = Volume of 30% Solution (litres)

$x + y = 10$ ①

$x(10\%) + y(30\%) = 10(15\%)$

$\Rightarrow x + 3y = 15$ ②

② − ①

$$\begin{array}{r} x + 3y = 15 \\ -x - y = -10 \\ \hline 2y = 5 \end{array}$$

$y = 5/2 = 2\frac{1}{2}$ litres

$x + 2\frac{1}{2} = 10$

$x = 7\frac{1}{2}$ litres