Section 2.1 Quadratic equations

Example 1

Use factors to solve (i) $x^2 - 5x - 6 = 0$ (ii) $y^2 - 5y = 0$ (iii) $4t^2 - 100 = 0$

(i) $x^2 - 5x - 6 = 0$ QUADRATIC

$(x-6)(x+5) = 0$

$x = 6 \mid x = -5$

(ii) $y(y-5) = 0$ H.C.F

$y = 0 \mid y = 5$

(iii) $4t^2 - 100 = 0$ Diff. 2 Squares

$(2t-10)(2t+10) = 0$

$t = 5 \mid t = -5$

Aside

Method

① FACTORISE

② SOLVE

OR

$4t^2 = 100$

$t^2 = 25$

$t = \pm\sqrt{25} = \pm 5$

Example 2

Solve $x - 6 = \frac{3}{x}$. (Note: It is not always obvious that we are dealing with an equation of the form $ax^2 + bx + c = 0$.)

multiply by x

$x^2 - 6x = 3$

$x^2 - 6x - 3 = 0$

$a = 1$

$b = -6$

$c = -3$

FACTOR method won't work

$$x = \frac{+6 \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)} = \frac{+6 \pm \sqrt{36+12}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$$

Aside

Multiply by LCM

IF $ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 3

Solve $x^4 + x^2 - 6 = 0$ for $x \in R$.

let $u = x^2$

$u^2 + u - 6 = 0$

$(u + 3)(u - 2) = 0$

$u = -3$ or 2

$x^2 = -3$
$x = \sqrt{-3}$
$= \sqrt{3}i$

$x^2 = 2$
$x = \sqrt{2}$

Aside

Substitution method

Example 4

Solve $2x + 3\sqrt{x} = 5$ for $x \in R$.

$3\sqrt{x} = 5 - 2x$

$(3\sqrt{x})^2 = (5 - 2x)^2$

$9x = 25 - 20x + 4x^2$

$4x^2 - 29x + 25 = 0$

$(x - 1)(4x - 25) = 0$

$x = 1$ | $x = 25/4$

Aside

- isolate surd
- square
- solve

Section 2.1 Quadratic equations

2. Use the quadratic formula to solve each of the following, giving your answers correct to one place of decimals:

(a) (i) $x^2 - 2x - 2 = 0$

$a = 1$
$b = -2$
$c = -2$

$$x = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{12}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$x = 1 + \sqrt{3}$
$= 2.7$

$x = 1 - \sqrt{3}$
$= -0.7$

3. Use the quadratic formula to solve each of the following, leaving your answers in surd form:

(a) (i) $3x^2 + 4x - 5 = 0$

$a = 3$
$b = 4$
$c = -5$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-5)}}{2(3)} = \frac{-4 \pm \sqrt{16 + 60}}{6}$$

$$= \frac{-4 \pm \sqrt{76}}{6} = \frac{-4 \pm 2\sqrt{19}}{6}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{19}}{3}$$

4. Solve the following equations:

(a) (i) $\frac{x+7}{3} + \frac{2}{x} = 4$

$x^2 + 7x + 6 = 12x$

$x^2 - 5x + 6 = 0$

$(x-6)(x+1) = 0$

$x = 6 \mid x = -1$

5. By finding a suitable substitution, solve each of the following:

(c) $\left(y + \frac{4}{y}\right)^2 - 9\left(y + \frac{4}{y}\right) + 20 = 0$

let $x = y + \frac{4}{y}$

$\Rightarrow x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x = 5 \mid x = 4$

Sub back

$\Rightarrow$

$5 = y + \frac{4}{y}$

multiply by y

$5y = y^2 + 4$

$y^2 - 5y + 4 = 0$

$(y-4)(y-1) = 0$

$y = 4 \mid y = 1$

$4 = y + \frac{4}{y}$

multiply by y

$4y = y^2 + 4$

$y^2 - 4y + 4 = 0$

$(y-2)(-2) = 0$

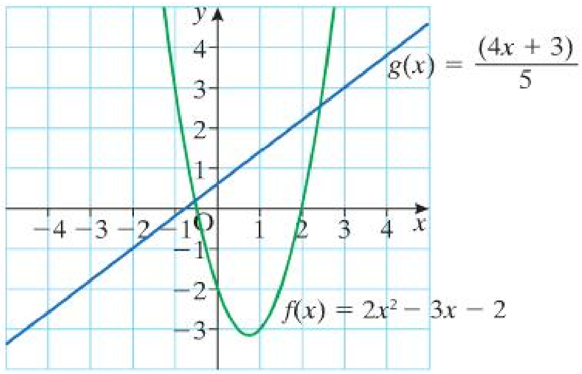
$y = 2 \mid y = 2$

10. The graphs of the functions

$f(x) = 2x^2 - 3x - 2$ and $g(x) = \frac{4x+3}{5}$

are drawn as shown. Using the graphs, estimate the solutions of the following equations

(a) $f(x) = 0$
(b) $g(x) = 0$
(c) $f(x) = g(x)$.



(a) $x = -0.5$ and $x = 2$

(b) $x = -0.7$

(c) $x = -0.6$ and $x \simeq 2.4$

Section 2.2 **Nature of quadratic roots**

Example 1

Evaluate the discriminant of each of the following, stating whether the equation has
(i) two distinct real roots (ii) two identical real roots (iii) no real roots.

(a) $3x^2 + 5x - 1 = 0$ (b) $49x^2 + 42x + 9 = 0$
(c) $2x^2 + 8x + 9 = 0$ (d) $2x^2 + 7x + 4 = 0$

a
$\Delta = (5)^2 - 4(3)(-1)$
$= 25 + 12 = 37 > 0$
2 Real

b
$\Delta = (42)^2 - 4(49)(9)$
$= 0$
2 identical

c
$\Delta = (8)^2 - 4(2)(9)$
$= 64 - 72$
$= -8 < 0$
no Real

d
$\Delta = (7)^2 - 4(2)(4)$
$= 49 - 32$
$= 17 > 0$
2 real

Aside

$\Delta = b^2 - 4ac$

$\Delta > 0$
2 real Roots

$\Delta = 0$
2 identical Real

$\Delta < 0$
2 imaginary Roots

Example 2

Find the values of k so that $-8 + kx - 2x^2 = 0$ has equal roots.

$$(k)^2 - 4(-8)(-2) = 0$$

$$K^2 - 64 = 0$$

$$K^2 = 64$$

$$K = \pm 8$$

Aside

$\Delta = 0$

$\Rightarrow$ equal Roots

$\Delta = b^2 - 4ac$

Example 3

Given the equation $px^2 + (p+q)x + q = 0$.

(i) Show that the roots are real for all values of p and $q \in R$.

(ii) Show that the roots are rational.

(iii) Hence find

(a) the roots, in terms of p and q

(b) the factors, in terms of p and q.

$px^2 + (p+q)x + q = 0 \rightarrow a = p, b = (p+q), c = q.$

(i)

$$\Delta = (p+q)^2 - 4(p)(q)$$

$$= p^2 + 2pq + q^2 - 4pq$$

$$= p^2 - 2pq + q^2$$

$$= (p-q)(p-q) = (p-q)^2 \geq 0$$

Aside

$\Delta \geq 0$

If Roots Real

$\Delta = b^2 - 4ac$

$(\text{any no.})^2 \geq 0$

Example 3

Given the equation $px^2 + (p+q)x + q = 0$.

(i) Show that the roots are real for all values of p and $q \in R$.

(ii) Show that the roots are rational.

(iii) Hence find

(a) the roots, in terms of p and q

(b) the factors, in terms of p and q.

$px^2 + (p+q)x + q = 0 \rightarrow a = p, b = (p+q), c = q.$

$\Delta = (p-q)^2 \Rightarrow$ Rational Roots

$px^2 + (p+q)x + q = 0$

Factors: $(px + q)(x + 1) = 0$

Roots are $x = -\frac{q}{p}$ | $x = -1$

Aside

Roots are Rational if they can be written as a fraction of 2 integers

this would happen if Δ = perfect Square