## Section 2.2 Nature of quadratic roots

**9.** Prove that the equation $(k-2)x^2 + 2x - k = 0$ has real roots, whatever the value of $k$

$\Delta = b^2 - 4ac$ when Real $\Delta \geq 0$

$$\Delta = (2)^2 - 4(k-2)(-k)$$
$$= 4 + 4k^2 - 8k$$
$$= (2k-2)(2k-2)$$
$$= (2k-2)^2 \geq 0$$

QED

**3.** Find the discriminant of each of the following equations and state if the roots are
(a) real and different (b) real and equal (c) imaginary.

(i) $2x^2 + x + 5 = 0$ (ii) $-2x^2 + 3x + 1 = 0$ (iii) $3x^2 + 2x - 1 = 0$
(iv) $-3 + 2x - x^2 = 0$ (v) $x^2 + 8x + 16 = 0$ (vi) $25 - 10x + x^2 = 0$

i)
$$\Delta = 1^2 + 4(2)(5)$$
$$= 1 + 40 = 41$$
$$> 0$$
Real and different

ii)
$$\Delta = 3^2 - 4(-2)(1)$$
$$= 9 + 8$$
$$= 17 > 0$$
Real and different

iii)
$$\Delta = 2^2 - 4(3)(-1)$$
$$= 4 + 12$$
$$= 16 > 0$$
Real and different

iv)
$$\Delta = (2)^2 - 4(-1)(-3)$$
$$= 4 - 12$$
$$= -8 < 0$$
imaginary

v)
$$\Delta = 8^2 - 4(1)(16)$$
$$= 64 - 64$$
$$= 0$$
Real and equal

vi)
$$\Delta = (-10)^2 - 4(1)(25)$$
$$= 100 - 100$$
$$= 0$$
Real and equal

**10.** Find the value of $k$ for which the equation $(k-2)x^2 + x(2k+1) + k = 0$ has equal roots.

$\Rightarrow \Delta = 0$

$\Rightarrow (2k+1)^2 - 4(k-2)(k) = 0$

$\cancel{4k^2} + 4k + 1 - \cancel{4k^2} + 8k = 0$

$12k + 1 = 0$

$k = -\frac{1}{12}$

**13.** Show that the equation $x^2 - 2px + 3p^2 + q^2 = 0$ cannot have real roots for $p, q \in R$.

If Real $\Delta \geq 0$

$\Delta = (-2p)^2 - 4(1)(3p^2 + q^2)$

$= 4p^2 - 12p^2 - 4q^2$

$= -8p^2 - 4q^2$

$= -4(2p^2 + q^2) < 0$

$\Rightarrow$ imaginary roots

**Section 2.3 Solving quadratic and linear equations**

## Example 1

Find the point(s) of intersection between

(i) $x - y = -1$ (ii) $x - y = 3$ and the curve $y = x^2 + 5x + 1$.

(i)

① $+y = +x+1$

② $x+1 = x^2+5x+1$

$x^2+4x = 0$

$x(x+4) = 0$

$x = 0 \mid x = -4$

③ $y = 0+1 = 1$

pt $(0,1)$

$y = -4+1 = -3$

pt $(-4,-3)$

ii

① $y = x-3$

② $x-3 = x^2+5x+1$

$x^2+4x+4 = 0$

$(x+2)(x+2) = 0$

$x = -2 \mid x = -2$

③ $y = -2-3 = -5$

pt $(-2,-5)$

Aside

① Rewrite linear

② Sub in & Solve

③ Sub Ans. into linear

## Example 2

Show that there are no point(s) of intersection between the line $x - y = 5$ and the curve $y = x^2 + 5x + 1$.

If no intersection ⇒ no real solutions to simultaneous equations

① $x-5 = y$

② $x-5 = x^2+5x+1$

$x^2+4x+6 = 0$

$x = \dfrac{-4 \pm \sqrt{(4)^2-4(1)(6)}}{2(1)}$

$= \dfrac{-4 \pm \sqrt{-8}}{2} = -2 \pm 2i$

③ $y = -2 \pm 2i - 5 = -7 \pm 2i$

Solutions $(-2-2i, -7-2i)$ and $(-2+2i, -7+2i)$

Aside

① Rewrite linear

② Sub and solve quadratic

③ Sub back into linear

**Section 2.3** **Solving quadratic and linear equations**

Solve: **12.** $x^2 + y^2 + 2x - 4y + 3 = 0$
$x - y + 3 = 0$

① $x = y - 3$

② $(y-3)^2 + y^2 + 2(y-3) - 4y + 3 = 0$

$\underline{y^2 - 6y + 9} + y^2 + 2y - 6 - 4y + 3 = 0$

$2y^2 - 8y + 6 = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3 \quad | \quad y = 1$

③ $x = 3 - 3 = 0$ pt $(0, 3)$
$x = 1 - 3 = -2$ pt $(-2, 1)$

Steps

① Rewrite linear

② Sub and solve

③ Sub back in

**Section 2.4** **Quadratic and linear equations in context**

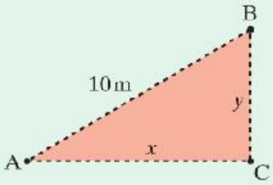
**Example 1**

A right-angled triangle is to be made from a rope 24 m long. If the hypotenuse of the triangle, AB, has to be 10 m, find

(i) an equation in terms of $x$ and $y$ for the perimeter of the triangle

(ii) an equation in terms of $x$ and $y$ for the hypotenuse of the triangle.

(iii) Solve the equations to find possible lengths of the base ($x$) and height ($y$) of the triangle.




(i) $24 = 10 + x + y$
$x + y = 14$

(ii) $10^2 = x^2 + y^2$
$100 = x^2 + y^2$

(iii) ① $x = 14 - y$

② $(14 - y)^2 + y^2 = 100$

$196 - 28y + y^2 + y^2 - 100 = 0$

$2y^2 - 28y + 96 = 0$

$y^2 - 14y + 48 = 0$

$(y - 6)(y - 8) = 0$

$y = 6 \quad | \quad y = 8$

③ $x = 14 - 6 = 8$
pt $(6, 8)$

$x = 14 - 8 = 6$
pt $(8, 6)$

Aside

Pytagoras

$a^2 = b^2 + c^2$

HW

Steps.

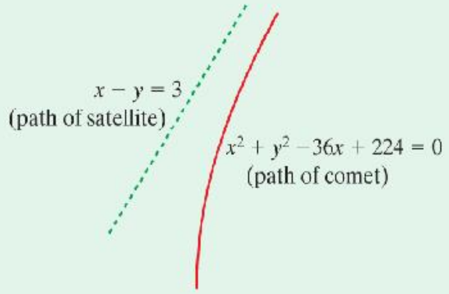
① Rewrite linear

② Sub & Solve quad.

③ Sub into linear

## Example 2

A satellite is on a fact-finding mission to the moons of Pluto. The equation $x - y = 3$ represents its path. A comet is discovered moving in a curve in the same plane as the satellite. If the path of the comet is determined to be $x^2 + y^2 - 36x + 224 = 0$, decide if their paths will cross.




① $x = 3 + y$

② $(3+y)^2 + y^2 - 36(3+y) + 224 = 0$

$9 + 6y + y^2 + y^2 - 72 - 36y + 224 = 0$

$2y^2 - 30y + 161 = 0$

$\Delta = (-30)^2 - 4(2)(161) > 0$

$\Rightarrow$ they will cross

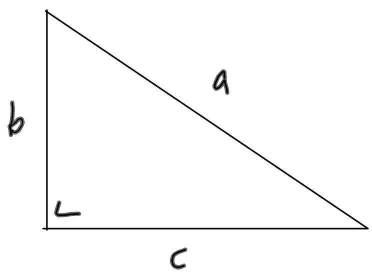
Aside

If they intersect then there is a real solution to the simultaneous equations

ie. $\Delta \geq 0$ of quadratic

**Section 2.4 Quadratic and linear equations in context**

**10.** The hypotenuse of a right-angled triangle is 6 cm longer than the shortest side. The third side is 3 cm longer than the shortest side. Find the length of the shortest side.




let $b$ = Shortest Side

$a = b + 6$

$c = b + 3$

$a^2 = b^2 + c^2$

$(b+6)^2 = b^2 + (b+3)^2$

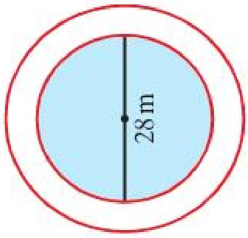
$b^2 + 12b + 36 = b^2 + b^2 + 6b + 9$

$b^2 - 6b - 27 = 0$

$(b - 9)(b + 3) = 0$

$b = 9$ ✓ | $b = -3$ ✗

**13.** A circular swimming pool with a diameter of 28 metres has a wooden deck around its edge.
If the deck has an area of $60\pi\,\text{m}^2$, find the width of the deck.




Area Circle $= \pi R^2$

let width path $= x$

Pool area $= \pi(14)^2 = 196\pi$

Total Area $= \pi(14+x)^2 = 196\pi + 60\pi$

$\Rightarrow \cancel{196} + 28x + x^2 = \cancel{196} + 60$

$x^2 + 28x - 60 = 0$

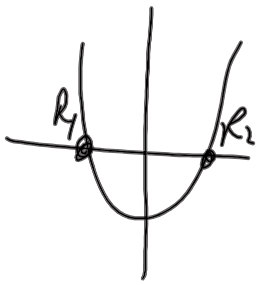
$(x+30)(x-2) = 0$

$x = -30$ ✗ | $x = 2$ ✓

# Roots of quadratic equations.

If $1x^2 + bx + c = 0$

$x^2 - \left(\text{sum of Roots}\right) + \left(\text{Product Roots}\right) = 0$

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eg .. $x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

Roots $x = -5$, $x = -1$

$R_1$ $R_2$

Sum of Roots = $R_1 + R_2 = -6$

Product of Roots = $(R_1)(R_2) = 5$

If $x^2$ cooefficient $\neq 1$

If $ax^2 + bx + c = 0$

then $\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$$\boxed{x^2 - (\text{Sum})x + (\text{PRODUCT}) = 0}$$

$$R_1 R_2 = \frac{c}{a}$$

$$R_1 + R_2 = \frac{-b}{a}$$

**Section 2.5** **Forming quadratic equations from their roots**

## Example 1

Write the equation of a curve whose roots are 7 and −5.

Sum Product

$$x^2 - (7-5)x + (7)(-5) = 0$$

$$x^2 - 2x - 35 = 0 \quad ✓$$

Aside

$x^2 - (\text{Sum})x + \text{Product} = 0$

$x^2 - (R_1 + R_2)x + (R_1 R_2) = 0$

## Example 2

If $x = \sqrt{3}$ and $x = \frac{-\sqrt{3}}{2}$ are the roots of a quadratic equation $ax^2 + bx + c = 0$, find $a, b$ and $c$.

$$x^2 - \left(\sqrt{3} + -\frac{\sqrt{3}}{2}\right)x + (\sqrt{3})\left(-\frac{\sqrt{3}}{2}\right) = 0$$

$$x^2 - \left(\frac{2\sqrt{3} - \sqrt{3}}{2}\right)x + \left(\frac{-3}{2}\right) = 0$$

$$x^2 - \frac{\sqrt{3}}{2}x - \frac{3}{2} = 0 \quad \checkmark$$

$$\Rightarrow \quad 2x^2 - \sqrt{3}x - 3 = 0$$

Aside

$$x^2 - (\text{Sum})x + \text{Product} = 0$$

**Section 2.5 Forming quadratic equations from their roots**

1. State (i) the sum and (ii) the product of the roots of each of the following quadratic equations.

(a) $x^2 + 9x + 4 = 0$ (b) $x^2 - 2x - 5 = 0$

(a)

Sum of Roots $= -9$

Product of Roots $= 4$

(b)

Sum of Roots $= 2$

Product of Roots $= -5$

**3.** Find the quadratic equations that have the following pairs of roots $(r_1, r_2)$.

(iv) $(\sqrt{5}, 4)$

$$x^2 - (R_1 + R_2)x + R_1R_2 = 0$$

$$x^2 - (4 + \sqrt{5})x + 4\sqrt{5} = 0$$

(viii) $\left(\frac{5}{2}, \frac{3}{5}\right)$

$$x^2 - (R_1 + R_2)x + R_1R_2 = 0$$

$$x^2 - \left(\frac{5}{2} + \frac{3}{5}\right)x + \left(\frac{5}{2}\right)\left(\frac{3}{5}\right) = 0$$

$$x^2 - \left(\frac{25+6}{10}\right)x + \frac{3}{2} = 0$$

$$x^2 - \frac{31}{10}x + \frac{3}{2} = 0$$

multiply by 10

$$10x^2 - 31x + 3 = 0$$