6th year honours maths Test on Algebra Chapter 2

Q.1 Show that the roots of the equation $x^2 - (a + d)x + (ad - b^2) = 0$ are real.

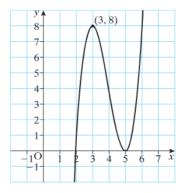
Q.2

A section of the graph of a polynomial

$$f(x) = ax^3 + bx^2 + cx + d$$

is drawn in this diagram.

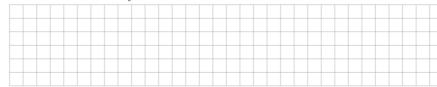
- (i) Find the roots of this polynomial.
- (ii) Write an expression for f(x) in terms of the factors of this polynomial.
- (iii) Find the values of a, b, c and d.
- (iv) Find an expression for the reflected image of this curve in the *x*-axis.
- (v) Find an expression for the reflected image of this curve in the *y*-axis.



Q.3

A cubic function f is defined for $x \in \mathbb{R}$ as $f: x \mapsto x^3 + (1-k^2)x + k \;, \quad \text{where } k \text{ is a constant}.$

(a) Show that -k is a root of f.



(b) Find, in terms of k, the other two roots of f.

- Q.4 Express $2x^2 4x 5$ in the form $a(x + h)^2 + k$ and hence,
 - (i) solve the equation $2x^2 4x 5 = 0$
 - (ii) find the minimum point of this curve.