5th Year honours maths
Test on Algebra Chapter 2

Question 1

By using the discriminant, determine the nature of the roots of each of the following:

(i) $x^2 - 2x - 5 = 0$ (ii) $x^2 - 4x + 6 = 0$ (iii) $-6 + 4x - x^2 = 0$

$\Delta = b^2 - 4ac$

(i) $\Delta = (-2)^2 - 4(1)(-5)$
$= 4 + 20 = 24 > 0$
$\Rightarrow$ 2 real roots

(ii) $\Delta = (-4)^2 - 4(1)(6)$
$= 16 - 24 = -8 < 0$
$\Rightarrow$ 2 imaginary roots

(iii) $\Delta = (4)^2 - 4(-1)(-6)$
$= 16 - 24 = -8 < 0$
$\Rightarrow$ 2 imaginary roots

Question 2

Using trial and error, find

(i) a root of the polynomial $f(x) = x^3 - 4x^2 - 11x + 30$
(ii) the factors of $f(x)$, and
(iii) hence solve the equation $x^3 - 4x^2 - 11x + 30 = 0$.

$f(2) = (2)^3 - 4(2)^2 - 11(2) + 30 = 8 - 16 - 22 + 30 = 0$
$\Rightarrow (x-2)$ is factor

$$x-2 \overline{)\, x^3 - 4x^2 - 11x + 30}$$
$$\mp x^3 \pm 2x^2$$
$$-2x^2 - 11x$$
$$\pm 2x^2 \mp 4x$$
$$-15x + 30$$
$$\pm 15x \mp 30$$

$x^2 - 2x - 15 = 0$
$(x+3)(x-5) = 0$
$x = -3$ or 5

FACTORS:
$(x-2)(x+3)(x-5)$

SOLUTIONS:
$x = 2, 5, -3$

Question 3

Express $2x^2 - 4x - 5$ in the form $a(x + h)^2 + k$ and hence,

(i) solve the equation $2x^2 - 4x - 5 = 0$

(ii) find the minimum point of this curve.

$2x^2 - 4x - 5 = 0 \Rightarrow 2\left[x^2 - 2x - \frac{5}{2}\right] = 0$

	x	-1
x	x^2	$-x$
-1	$-x$	$+1$

$2\left[x^2 - 2x + 1 - 1 - \frac{5}{2}\right] = 0$

$2\left[(x-1)^2 - \frac{7}{2}\right] = 0$

$2(x-1)^2 - 7 = 0$

(ii) min: $(1, -7)$

(i) Hence solve $2(x-1)^2 - 7 = 0$

$2(x-1)^2 = 7$

$(x-1)^2 = \frac{7}{2}$

$x - 1 = \pm\sqrt{\frac{7}{2}}$

$x = 1 \pm \sqrt{\frac{7}{2}}$

[note this is the same as $x = \frac{2 \pm \sqrt{14}}{2}$]

Question 4

a) Show that $\dfrac{-1+\sqrt{3}}{1+\sqrt{3}} = 2-\sqrt{3}$.

$$\frac{(-1+\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{-1+\sqrt{3}+\sqrt{3}-3}{1-3}$$

$$= \frac{-4+2\sqrt{3}}{-2} = 2-\sqrt{3} \quad \text{RED}$$

b) Solve each of these equations and check each solution:

(i) $\left(\sqrt{x+5}\right)^2 = \left(5-\sqrt{x}\right)^2$

$$x+5 = 25 - 10\sqrt{x} + x$$

$$-20 = -10\sqrt{x}$$

$$2 = \sqrt{x}$$

$$x = 4$$

Check $\left(\sqrt{4+5}\right) \stackrel{?}{=} 5-\sqrt{4}$

$\sqrt{9} \stackrel{?}{=} 5-2$

$3 \stackrel{?}{=} 3$ ✓ yes

Question 5

If r_1 and r_2 are the roots of the equation $x^2 - \sqrt{3}x - 6 = 0$, evaluate $r_1 r_2$.

$r_1 r_2 = -6$

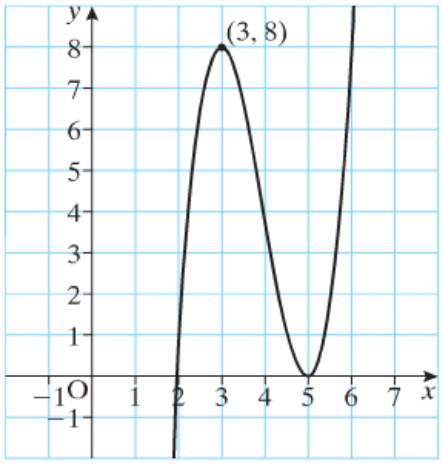
Question 6

A section of the graph of a polynomial

$$f(x) = ax^3 + bx^2 + cx + d$$

is drawn in this diagram.

(i) Find the roots of this polynomial.
(ii) Write an expression for $f(x)$ in terms of the factors of this polynomial.
(iii) Find the values of a, b, c and d.
(iv) Find an expression for the reflected image of this curve in the x-axis.
(v) Find an expression for the reflected image of this curve in the y-axis.



Roots: $x = 2, 5$

FACTORS: $(x-2)(x-5)(x-5)$