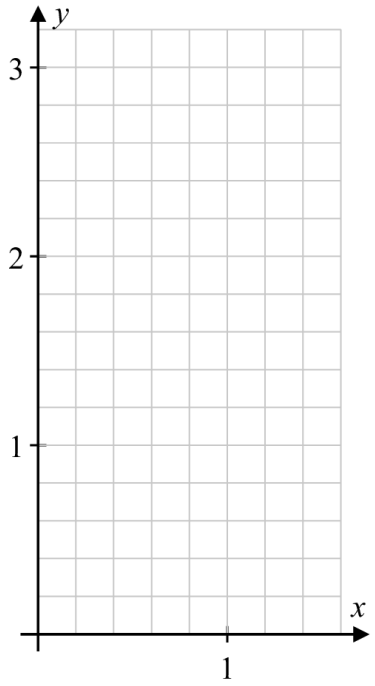
(a) **(i)** $f(x) = \frac{2}{e^x}$ and $g(x) = e^x - 1$, where $x \in \mathbb{R}$.

Complete the table below. Write your values correct to two decimal places where necessary.

x	**0**	**0·5**	**1**	**ln(4)**
$f(x) = \frac{2}{e^x}$				
$g(x) = e^x - 1$				

(ii) In the grid on the right, use the table to draw the graphs of $f(x)$ and $g(x)$ in the domain $0 \leq x \leq \ln(4)$. Label each graph clearly.



(iii) Use your graphs to estimate the value of x for which $f(x) = g(x)$.

(b) Solve $f(x) = g(x)$ using algebra.

Question 1 **(25 marks)**

(a) Solve the simultaneous equations.

$$\begin{aligned} 2x + 3y - z &= -4 \\ 3x + 2y + 2z &= 14 \\ x - 3z &= -13 \end{aligned}$$

(b) Solve the inequality $\frac{2x-3}{x+2} \geq 3$, where $x \in \mathbb{R}$ and $x \neq -2$.

(a) Solve the equation $x = \sqrt{x+6},\ x \in \mathbb{R}$.

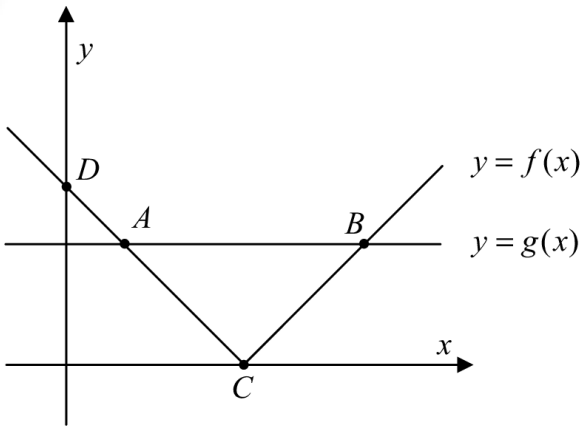
Question 4 **(25 marks)**

(a) Solve the simultaneous equations:

$$2x+8y-3z=-1$$
$$2x-3y+2z=2$$
$$2x+y+z=5.$$

(b) The graphs of the functions $f:x\mapsto|x-3|$ and $g:x\mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.



$A=(\quad,\quad)$ $B=(\quad,\quad)$

$C=(\quad,\quad)$ $D=(\quad,\quad)$

(ii) Hence, or otherwise, solve the inequality $|x-3|<2$.

Question 1 **(25 marks)**

(a) Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$
$$a + 2b + 1 = 0$$

(b) Find the set of all real values of x for which $\frac{2x-5}{x-3} \leq \frac{5}{2}$.

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Question 7 **(55 marks)**

Sometimes it is possible to predict the future population in a city using a function.
The population in Sapphire City, over time, can be predicted using the following function:

$$p(t) = Se^{0{\cdot}1t} \times 10^6.$$

The population in Avalon, over time, can be predicted using the following function:

$$q(t) = 3{\cdot}9e^{kt} \times 10^6.$$

In the functions above, t is time, in years; $t = 0$ is the beginning of 2010; and both S and k are constants.

(a) The population in Sapphire City at the beginning of 2010 is 1 100 000 people.
Find the value of S.

(b) Find the predicted population in Sapphire City at the beginning of 2015.

(c) Find the predicted change in the population in Sapphire City during 2015.

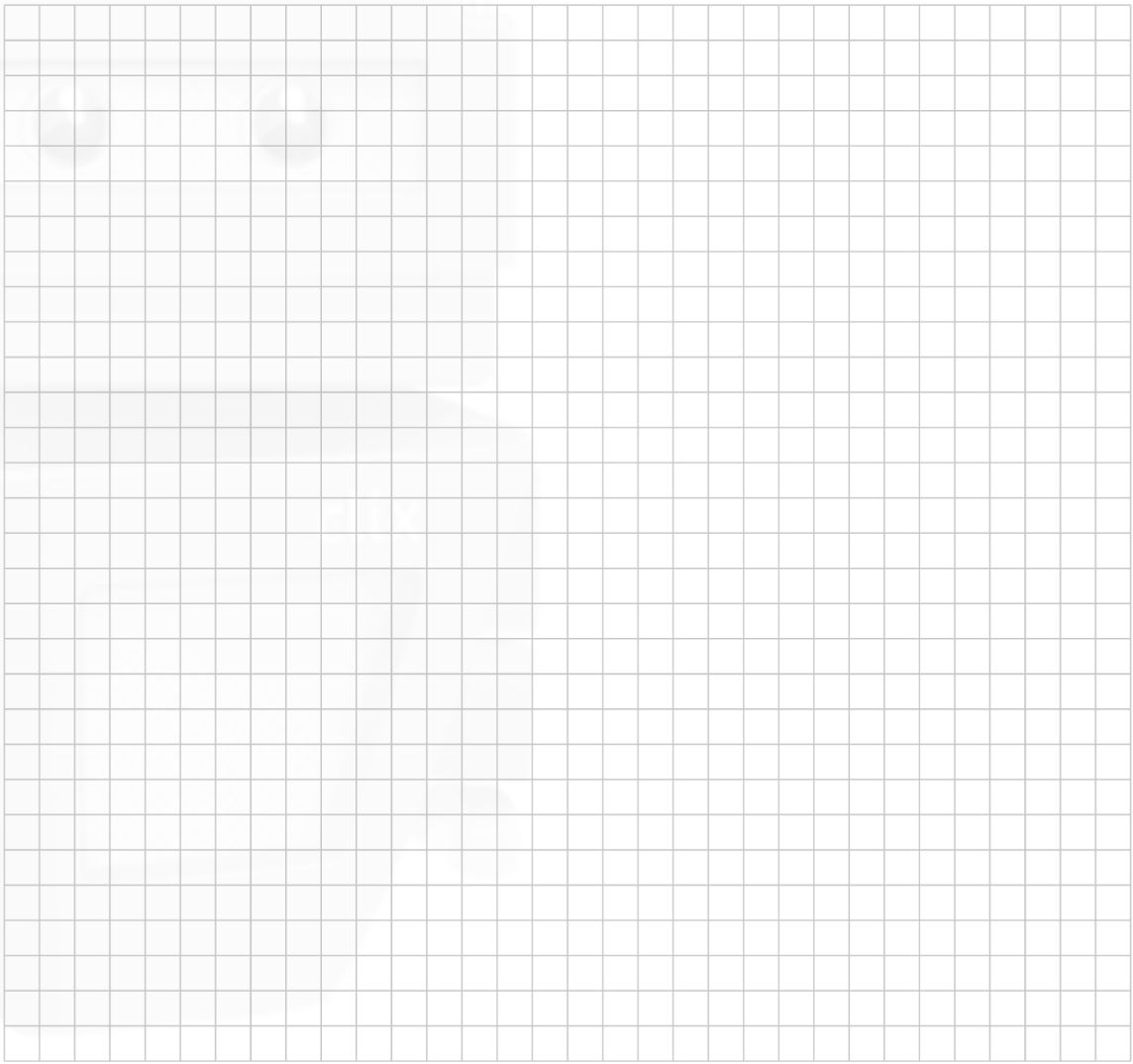
(d) The predicted population in Avalon at the beginning of 2011 is 3 709 795 people.
Write down and solve an equation in k to show that $k = -0{\cdot}05$, correct to 2 decimal places.

(e) Find the year during which the populations in both cities will be equal.

(f) Find the predicted average population in Avalon from the beginning of 2010 to the beginning of 2025.

This question is continued on the next page

(g) Use the function $q(t) = 3{\cdot}9e^{-0{\cdot}05t} \times 10^6$ to find the predicted rate of change of the population in Avalon at the beginning of 2018.



(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

(ii) $\log_a \frac{9a^2}{16}$.

Scientists can estimate the age of certain ancient items by measuring the proportion of carbon−14, relative to the total carbon content in the item. The formula used is $Q = e^{-\frac{0{\cdot}693t}{5730}}$, where Q is the proportion of carbon−14 remaining and t is the age, in years, of the item.

(a) An item is 2000 years old. Use the formula to find the proportion of carbon−14 in the item.

(b) The proportion of carbon−14 in an item found at Lough Boora, County Offaly, was 0·3402. Estimate, correct to two significant figures, the age of the item.

(b) Given that $p = \log_c x$, express $\log_c \sqrt{x} + \log_c (cx)$ in terms of p.

Question 7 **(55 marks)**

The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function:

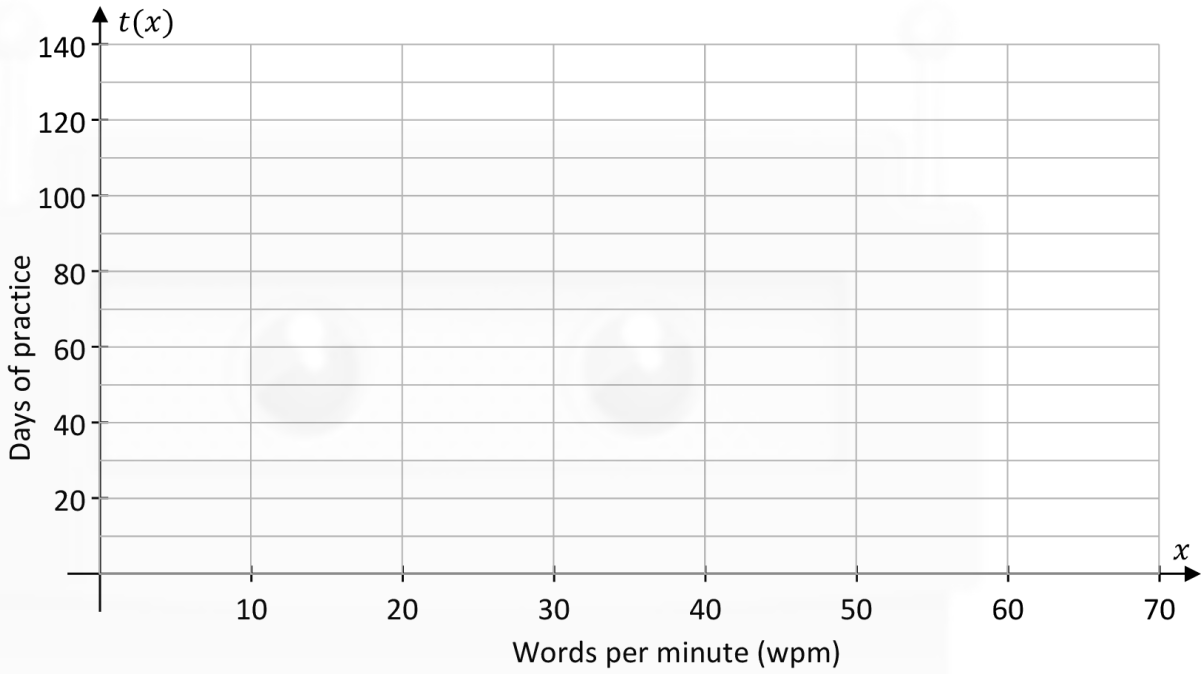
$$t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right], \text{ where } 0 \leq x \leq 70, x \in \mathbb{R}, \text{ and } k \text{ is a constant.}$$

(a) Based on the function $t(x)$, Jack can learn to type 35 wpm in 35·96 days. Write the function above in terms of k and **hence** show that $k = -62{\cdot}5$, correct to 1 decimal place.

(b) Find the number of wpm that Jack can learn to type with 100 days of practice.
Give your answer correct to the nearest whole number.

(c) Complete the table below, correct to the nearest whole number **and hence** draw the graph of $t(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)								



(d) A simpler function that could also be used to model the number of days needed to attain x wpm is $p(x) = 1{\cdot}5x$.
Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.

(e) Let $h(x) = p(x) - t(x)$.

(i) Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

(ii) Use calculus to find the maximum value of $h(x)$ for $0 \leq x \leq 70,\ x \in \mathbb{R}$.
Give your answer correct to the nearest whole number.

Question 4 **(25 marks)**

(a) The amount of a substance remaining in a solution reduces exponentially over time. An experiment measures the percentage of the substance remaining in the solution. The percentage is measured at the same time each day. The data collected over the first 4 days are given in the table below. Based on the data in the table, estimate which is the first day on which the percentage of the substance in the solution will be less than 0·01%.

Day	1	2	3	4
Percentage of substance (%)	95	42·75	19·2375	8·6569

(a) Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$.

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$