1 (2016)

Q3	Model Solution – 25 Marks	Marking Notes

(a)

(i)

x	0	0·5	1	$\ln(4)$
$f(x)=\dfrac{2}{e^x}$	2	1·21	0·74	0·5
$g(x)=e^x-1$	0	0·65	1·72	3

Scale 5C (0, 2, 4, 5)

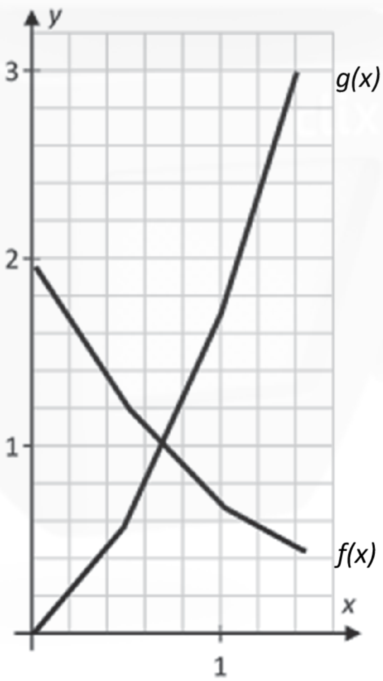
Low Partial Credit

- one entry correct

High Partial Credit

- 5 entries correct

(ii)



Scale 5C (0, 2, 4, 5)

Low Partial Credit

- one plot correct

High Partial Credit

- 5 plots correct
- one correct graph
- no labelling

Notes:

- *straight lines NOT acceptable*
- *one clear label merits full credit*
- *one ambiguous label merits High Partial Credit at most*

(iii)

$$f(x)=g(x) \text{ when } x \approx 0\cdot 7$$

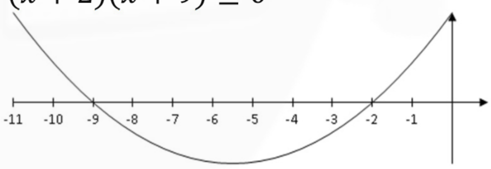
Scale 5B (0, 2, 5)

Partial Credit

- point of intersection clearly indicated on graph, but value of x not stated

Q3	Model Solution – Continued	Marking Notes
(b)	$\frac{e^x - 1}{1} = \frac{2}{e^x}$ $e^{2x} - e^x = 2$ $(e^x)^2 - e^x - 2 = 0$ $(e^x - 2)(e^x + 1) = 0$ $e^x = 2$ **or** $e^x = -1$ $x = \ln 2$ or $x = 0{\cdot}693$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • substitution correct *High Partial Credit* • correct factors of quadratic • root formula correctly substituted $e^x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ **Note:** *oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most*
	Or	**Or**
	$(e^x)^2 - e^x - 2 = 0$ Let $y = e^x \Rightarrow y^2 - y - 2 = 0$ $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ $= \frac{1 \pm \sqrt{1+8}}{2}$ $= \frac{1 \pm 3}{2}$ $\Rightarrow y = 2$ or $y = -1$ (not possible) $y = e^x \Rightarrow e^x = 2$ $x = \ln 2$ or $x = 0{\cdot}693$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • substitution correct *High Partial Credit* • root formula correctly substituted $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ **Note:** *oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most*

2 (2018)

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$(i)\quad 2x+3y-z=-4 \qquad \times(2)$ $(ii)\quad 3x+2y+2z=14 \qquad \times(-3)$ $4x+6y-2z=-8$ $-9x-6y-6z=-42$ $-5x-8z=-50$ $(iii)\quad x-3z=-13 \qquad \times(5)$ $-5x-8z=-50$ $5x-15z=-65$ $-23z=-115$ $z=5$ $\Rightarrow x=2$ $\Rightarrow y=-1 \qquad \{2,-1,5\}$	**Scale 15D (0, 5, 7, 11, 15)** *Low Partial Credit:* Matches coefficient of 1 variable in 2 equations Writes x in terms of z in eq (*iii*) *Mid Partial Credit:* 1 unknown found with errors Eliminates one unknown 1 unknown found and stops *High Partial Credit:* 2 unknowns found
(b)	$\dfrac{2x-3}{x+2}\geq 3 \qquad \times\ (x+2)^2$ $(2x-3)(x+2)\geq 3(x+2)^2$ $2x^2+x-6\geq 3x^2+12x+12$ $x^2+11x+18\leq 0$ $(x+2)(x+9)\leq 0$  $-9\leq x<-2$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit* Use of $(x+2)^2$ Relevant work but with linear inequality Squares both sides with some subsequent work (low partial credit at most) *Mid Partial Credit:* Quadratic inequality involving 0 *High Partial Credit* Roots of quadratic found **Note**: Accept $-9\leq x\leq -2$

3 (2015)

$$x=\sqrt{x+6}$$
$$\Rightarrow x^2=x+6$$
$$\Rightarrow x^2-x-6=0$$
$$\Rightarrow (x+2)(x-3)=0$$
$$\Rightarrow x=-2,\quad x=3$$
$$x=-2:\quad -2\neq\sqrt{-2+6}=\sqrt{4}=2 \ \text{✗}$$
$$x=3:\quad 3=\sqrt{3+6}=\sqrt{9}=3 \ \checkmark$$

We can subtract the second equation from the first:

$$\begin{array}{rcl} 2x+8y-3z &=& -1 \\ 2x-3y+2z &=& 2 \\ \hline 11y-5z &=& -3 \end{array}$$

Similarly, we subtract the third equation from the first:

$$\begin{array}{rcl} 2x+8y-3z &=& -1 \\ 2x+y+z &=& 5 \\ \hline 7y-4z &=& -6 \end{array}$$

Now we solve the simultaneous equations

$$\begin{array}{rcl} 11y-5z &=& -3 \\ 7y-4z &=& -6 \end{array}$$

Multiply the first by 7, the second by 11 and subtract:

$$\begin{array}{rcl} 77y-35z &=& -21 \\ 77y-44z &=& -66 \\ \hline 9z &=& 45 \end{array}$$

Therefore $z = \frac{45}{9} = 5$. Now substitute $z = 5$ into $7y-4z = -6$ to get $7y-4(5) = -6$ or $7y = -6+20 = 14$ Therefore $y = 2$.
Finally substitute $y = 2$ and $z = 5$ into $2x+8y-3z = -1$ to get $2x+8(2)-3(5) = -1$ or $2x = -1-8(2)+3(5) = -2$ So $x = -1$.
So the solution is

$$x = -1, y = 2, z = 5.$$

Now we can check this by substituting into the original equations and verifying that they are all true:

$$\begin{array}{rcl} 2(-1)+8(2)-3(5) &=& -1 \\ 2(-1)-3(2)+2(5) &=& 2 \\ 2(-1)+(2)+(5) &=& 5. \end{array}$$

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(b) The graphs of the functions $f : x \mapsto |x-3|$ and $g : x \mapsto 2$ are shown in the diagram.

(i) Find the co-ordinates of the points A, B, C and D.

D is on the y-axis, so its x-co-ordinate is 0. Now $f(0) = |0-3| = |-3| = 3$. So $D = (0,3)$.
$C = (,0)$ (on the x-axis), so we solve $|x-3| = 0$ to find the x-co-ordinate. Now $|x-3| = 0 \Leftrightarrow x-3 = 0 \Leftrightarrow x = 3$. So $C = (3,0)$.
A and B both have y-co-ordinate 2, so we solve $|x-3| = 2$. Now $|x-3| = 2 \Leftrightarrow \pm(x-3) = 2$. So either

$$(x-3) = 2 \text{ or } -(x-3) = 2.$$

In the first case $x = 5$ and in the second case $-x+3 = 2$ or $x = 1$. So $A = (1,2)$ and $B = (5,2)$.

$$A = (1,2) \quad B = (5,2)$$
$$C = (3,0) \quad D = (0,3)$$

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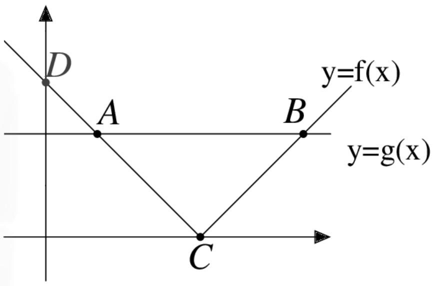

(ii) Hence, or otherwise, solve the inequality $|x-3| < 2$.

The solution set of the inequality corresponds to the values of x for which the graph of f is below the graph of g. From the diagram and calculations above, we see that the solution set is

$$1 < x < 5.$$

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$$a = -2b - 1$$

$$(-2b-1)^2 + (2b+1)b + b^2 = 3$$
$$7b^2 + 5b - 2 = 0$$
$$(7b-2)(b+1) = 0$$
$$b = \tfrac{2}{7} \quad \text{or} \quad b = -1$$
$$a = \tfrac{-11}{7} \quad \text{or} \quad a = 1$$

Solution: $\{b = \tfrac{2}{7}$ and $a = \tfrac{-11}{7}\}$ or $\{b = -1$ and $a = 1\}$.

(b) Find the set of all real values of x for which $\dfrac{2x-5}{x-3} \le \dfrac{5}{2}$.

Multiply across by $2(x-3)^2$, which is non-negative:

$$2(x-3)(2x-5) \le 5(x-3)^2$$
$$4x^2 - 22x + 30 \le 5x^2 - 30x + 45$$
$$0 \le x^2 - 8x + 15$$
$$0 \le (x-5)(x-3)$$

$x \ge 5$ or $x < 3$.

OR

$$\frac{2x-5}{x-3} - \frac{5}{2} \le 0$$
$$\frac{2(2x-5) - 5(x-3)}{2(x-3)} \le 0$$
$$\frac{-x+5}{2(x-3)} \le 0$$

	$x<3$	$3<x<5$	$x>5$
$-x+5$	+	+	–
$x-3$	–	+	+
$\dfrac{-x+5}{2(x-3)}$	–	+	–

$x \ge 5$ or $x < 3$.

(a)	$Se^{\cdot 1(0)} \times 10^6 = 1100000$ $S = 1{\cdot}1$	**Scale 10B (0, 4, 10)** *Partial Credit* • equation in S with substitution
(b)	$p(5) = 1{\cdot}1e^{0\cdot 1(5)} \times 10^6$ $= 1{\cdot}813593 \times 10^6$ $= 1813593$	**Scale 10B (0, 4, 10)** *Partial Credit* • substitution into formula for $p(5)$
(c)	$p(6) = 1{\cdot}1e^{0\cdot 6} \times 10^6$ $p(5) = 1{\cdot}1e^{0\cdot 5} \times 10^6$ $p(6) - p(5) = (1{\cdot}1e^{0\cdot 6} - 1{\cdot}1e^{0\cdot 5}) \times 10^6$ $= 0{\cdot}1907372 \times 10^6$ $= 190737$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • substitution into formula for $p(6)$ • use of $p(5)$ from previous part • $p(6) - p(5)$ written or implied *High partial Credit* • Formulates $p(6) - p(5)$ with some substitution

(d)	$q(t) = 3{\cdot}9e^{kt} \times 10^6$ $3709795 = 3{\cdot}9e^{k} \times 10^6$ $\frac{3{\cdot}709795}{3{\cdot}9} = e^k$ $\log_e \frac{3{\cdot}709795}{3{\cdot}9} = k$ $k = -0{\cdot}0499 = -0{\cdot}05$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit* • Either substitution into formula for k • Verifies k value only. *High Partial Credit* • relevant equation in k
(e)	$p(t) = q(t)$ $1{\cdot}1e^{0{\cdot}1t} \times 10^6 = 3{\cdot}9e^{-0{\cdot}05t} \times 10^6$ $1{\cdot}1e^{0{\cdot}1t} = 3{\cdot}9e^{-0{\cdot}05t}$ $\frac{e^{0{\cdot}1t}}{e^{-0{\cdot}05t}} = \frac{3{\cdot}9}{1{\cdot}1}$ $e^{0{\cdot}15t} = \frac{39}{11}$ $\ln\frac{39}{11} = 0{\cdot}15t$ $t = 8{\cdot}44$ years In 2018 both populations equal	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit* • $p(t) = q(t)$ written or implied *High Partial Credit* • relevant equation in t
(f)	$\frac{1}{15}\int_0^{15} 3{\cdot}9e^{-0{\cdot}05t} \times 10^6 dt$ $\frac{1}{15}\left[\frac{3{\cdot}9}{-0{\cdot}05}e^{-0{\cdot}05(15)} - \frac{3{\cdot}9}{-0{\cdot}05}e^{-0{\cdot}05(0)}\right]$ $\times 10^6$ $2{\cdot}743694 \times 10^6$ 2743694	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • integral formulated (with limits) *High Partial Credit:* • integration with full substitution
(g)	$q(t) = 3.9e^{-0.05t} \times 10^6$ $q'(t) = -0{\cdot}05(3{\cdot}9e^{-0{\cdot}05t} \times 10^6)$ $q'(8) = -0{\cdot}05\left(3{\cdot}9e^{-0{\cdot}05(8)} \times 10^6\right)$ $= -130712$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit* • $q'(t)$ *High Partial Credit* • $q'(t)$ fully substituted

7 (2016)

(b) **(i)**	$p = \log_a 2$, $q = \log_a 3$ $\log_a \frac{8}{3} = \log_a 8 - \log_a 3$ $= \log_a (2)^3 - \log_a 3$ $= 3\log_a 2 - \log_a 3$ $= 3p - q$	Scale 5C (0, 2, 4, 5) *low Partial Credit* • $\log_a 8 - \log_a 3$ *High Partial Credit* • $\log_a 8 = 3\log_a 2$ (and/or $= 3p$)
(ii)	$\log_a \frac{9a^2}{16} = \log_a (3a)^2 - \log_a (2)^4$ $= 2\log_a 3 + 2\log_a a - 4\log_a 2$ $= 2q + 2(1) - 4p$ $= 2q + 2 - 4p$	Scale 5D (0, 2, 3, 4, 5) *Low Partial Credit* • $\log_a 9a^2 - \log_a 16$ *Mid Partial Credit* • $2\log_a 3$ • $2\log_a a$ • $4\log_a 2$ • $4p$ or $2q$ or 2 *High Partial Credit* • $2(\log_a 3 + \log_a a) - 4\log_a 2$ or equivalent

8 (2013)

$$Q = e^{-\frac{0\cdot693t}{5730}} = e^{-\frac{0\cdot693\times2000}{5730}} = 0\cdot7851$$

$$Q = e^{-\frac{0\cdot693t}{5730}} = 0\cdot3402$$

$$\Rightarrow -\frac{0\cdot693t}{5730} = \ln 0\cdot3402$$

$$\Rightarrow t = -\frac{5730 \times \ln 0\cdot3402}{0\cdot693} \approx 8915 \approx 8900 \text{ years}$$

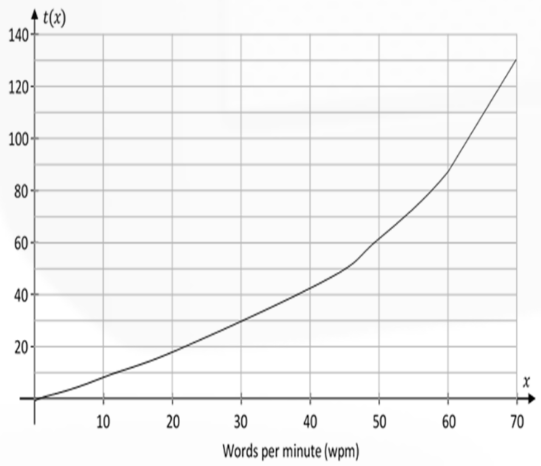
9 (2012)

Q7	Model Solution – 55 Marks	Marking Notes
(a)	$35{\cdot}96 = k\ln\left(1-\frac{35}{80}\right)$ $35{\cdot}96 = k\ln\left(\frac{45}{80}\right)$ $k = \dfrac{35{\cdot}96}{\ln\left(\frac{45}{80}\right)}$ $k = -62{\cdot}5$ to one place of decimals	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Effort at transposing Some substitution into function Full substitution and stops *High Partial Credit:* Function written in terms of k and fully substituted One incorrect substitution worked correctly and with some reference to $k \neq -62{\cdot}5$
(b)	$100 = -62{\cdot}5\ln\left(1-\frac{x}{80}\right)$ $\frac{100}{-62{\cdot}5} = \ln\left(1-\frac{x}{80}\right)$ $e^{\frac{100}{-62\cdot5}} = 1-\frac{x}{80}$ $x = -80(e^{\frac{100}{-62\cdot5}}-1)$ $x = 64$ wpm (To the nearest whole number)	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some substitution into function Trial and improvement (more than 1 iteration) Correct answer without work *High Partial Credit:* $e^{\frac{100}{-62\cdot5}} = 1-\frac{x}{80}$ Equation rewritten in terms of x or $\frac{x}{80} =$

(c)

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)	0	8	18	29	43	61	87	130

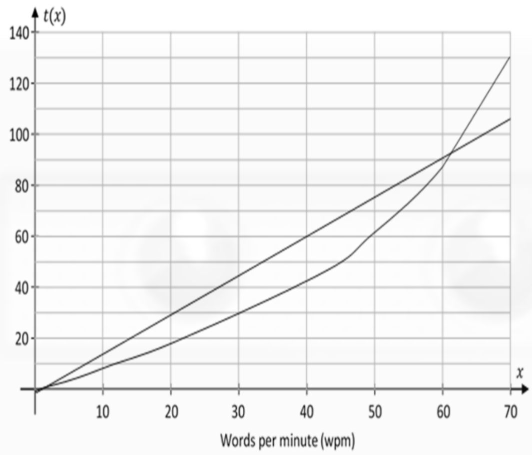
(c)



Scale 20D (0, 5, 10, 15, 20)
Low Partial Credit:
One entry correct
One plot (from candidates table) correct

Mid Partial Credit:
4 entries correct and 4 plots of table values

High Partial Credit:
All plots consistent with candidates table values (with at least 1 correct value)
Table correct but incorrect plots

<table>
<tr><td>(d)</td><td>
</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
One point on line identified
One point (not origin) plotted

High Partial Credit:
2 points on line identified and plotted</td></tr>
<tr><td>(e)
(i)</td><td>Approx 62 wpm</td><td>Scale 5B(0, 2, 5)
Partial Credit:
Point of intersection indicated on graph
$h(x)$ written in terms of x

Tolerance: $\pm$ 2 wpm</td></tr>
<tr><td>(e)
(ii)</td><td>For Maximum Value:
Set $h'(x) = 0$
$h(x) = 1{\cdot}5x + 62{\cdot}5 \ln\left(1 - \frac{x}{80}\right)$

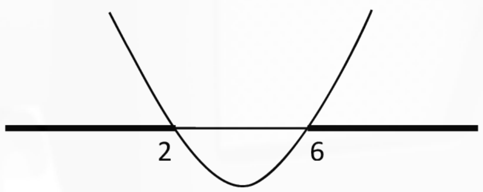
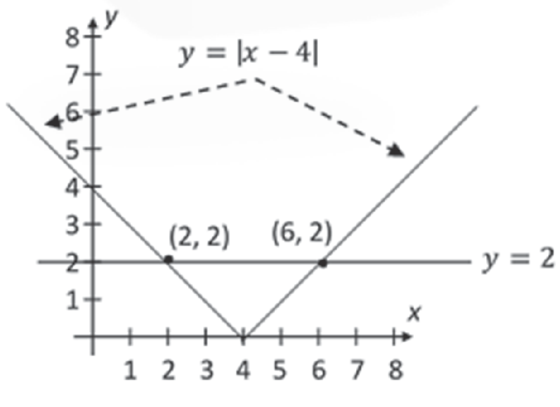
$h'(x) = 1{\cdot}5 + 62{\cdot}5\left(\frac{1}{1 - \frac{x}{80}}\right) \times \left(-\frac{1}{80}\right)$
$= 0$
$\frac{62{\cdot}5}{80 - x} = 1{\cdot}5$
$x = 80 - \frac{62{\cdot}5}{1{\cdot}5}$
$x = 38{\cdot}3 = 38 \text{ words}$

$h\left(38\frac{1}{3}\right) = 1{\cdot}5\left(38\frac{1}{3}\right)$
$+ 62{\cdot}5 \ln\left(1 - \frac{38\frac{1}{3}}{80}\right) = 16{\cdot}73$

$=$ 17 days</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
Any correct differentiation
$h(x) = 1{\cdot}5x + 62{\cdot}5 \ln\left(1 - \frac{x}{80}\right)$
$h'(x) = 0$

High Partial Credit:
Differentiation correct but un-simplified
Value for x and stops</td></tr>
</table>

(a)	$r = \frac{42\cdot75}{95} = \frac{9}{20}$ $\quad T_n = ar^{n-1} < 0\cdot01$ $$95\left(\frac{9}{20}\right)^{n-1} < 0\cdot01$$ $$\left(\frac{9}{20}\right)^{n-1} < \frac{0\cdot01}{95}$$ $$(n-1)\log\left(\frac{9}{20}\right) < \log\left(\frac{0.01}{95}\right)$$ $$(n-1) > \frac{\log\left(\frac{0.01}{95}\right)}{\log\left(\frac{9}{20}\right)}$$ (since $\log\left(\frac{9}{20}\right)$ is negative) $$n-1 > 11\cdot47$$ $$n > 12\cdot47$$ 12th day	**Scale 15D (0, 5, 8, 12, 15)** *Low Partial Credit:* • r found • T_n of a GP with some substitution *Mid Partial Credit:* • Inequality in n written *High Partial Credit:* • Inequality in n simplified (log handled) *Full Credit:* • Accept $n = 12\cdot47$

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$x^2 - 8x + 16 \geq 4$ $x^2 - 8x + 12 \geq 0$ $(x-2)(x-6) \geq 0$ $x = 2 \quad x = 6$ $\{x \mid x \leq 2\} \cup \{x \mid x \geq 6\}$ **Or** $x - 4 \geq 2 \ \cup x - 4 \leq -2$ $x \geq 6 \ \cup x \leq 2$ **Or** Graphical method (must indicate range on X-axis somehow) 	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • either side squared • one correct linear inequality written • stating range of natural numbers only *High Partial Credit:* • correct solutions to quadratic *Full Credit:* • correct answer without work **Note:** *use of natural numbers in range merits High Partial Credit at most*
	Or  $x \leq 2 \cup x \geq 6$	**Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • any one straight line *High Partial Credit:* • three straight lines

(b)	$$x = \frac{-3y-1}{2}$$ $$\left(\frac{-3y-1}{2}\right)^2 + \left(\frac{-3y-1}{2}\right)(y) + 2y^2 = 4$$ $$11y^2 + 4y - 15 = 0$$ $$(11y+15)(y-1) = 0$$ $$y = \frac{-15}{11} \;\; or \;\; y = 1$$ $$x = \frac{-3\left(\frac{-15}{11}\right) - 1}{2} \quad or \; x = \frac{-3(1) - 1}{2}$$ $$x = \frac{17}{11} \quad or \; x = -2$$ **or** $$y = \frac{-2x-1}{3}$$ $$x^2 + x\left(\frac{-2x-1}{3}\right) + 2\left(\frac{-2x-1}{3}\right)^2 = 4$$ $$11x^2 + 5x - 34 = 0$$ $$(11x-17)(x+2) = 0$$ $$x = \frac{17}{11} \quad \text{or } x = -2$$ $$y = \frac{-15}{11} \;\; \text{or} \;\; y = 1$$	Scale 15C (0, 5, 10,15) *Low Partial Credit:* • effort to isolate x (or y) *High Partial Credit:* • fully correct substitution into quadratic