In today's class we looked at questions in Algebra Sections 7.1 to 7.7

We also looked at the examples from Section 7.5 Regarding proofs involving inequalities.

(Video linked on Algebra 3 Page of website)

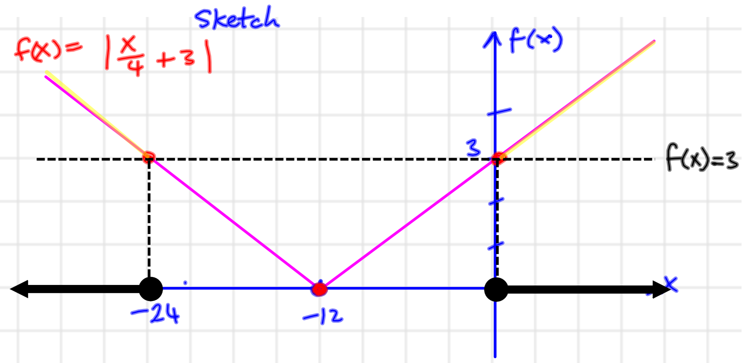
(7.3) **9.** Sketch the graph of the function $f(x) = |\frac{1}{4}x + 3|$ and hence find the solution to the inequality $|\frac{1}{4}x + 3| \geqslant 3$.

Recognise that this is a linear modulus ⇒ graph is a "V" shape bouncing off the x-axis

Sketch

$f(x) = |\frac{x}{4} + 3|$

x	f(x)
−12	0
0	3
−24	3



$\therefore \; |\frac{1}{4}x + 3| \geq 3 \quad \Rightarrow \quad -24 \geq x \geq 0$

algebraic method

$|\frac{1}{4}x + 3| \geq 3$

square: $(\frac{1}{4}x + 3)^2 \geq 3^2$

−9: $\frac{1}{16}x^2 + \frac{6}{4}x + \cancel{9} \geq \cancel{9}\,0$

×16: $x^2 + 24x \geq 0$

factors: $x(x+24) \geq 0$

Solve if $= 0$: if $x(x+24) = 0 \Rightarrow x = 0$ or -24

test $x=1$: $(1)^2 + 24(1) \overset{?}{\geq} 0$ true

Conclude: Since 1 is outside range −24 to 0 $\Rightarrow -24 \geq x \geq 0$

(7.6)

7. Simplify each of the following, writing your answers with positive indices.

(i) $\frac{(xy^2)^3 \times (x^2y)^{-2}}{xy}$ (ii) $\left(\frac{p^2q}{p^{-1}q^3}\right)^4$ (iii) $a^{\frac{1}{4}} \times a^{-\frac{5}{4}}$

addition Rule

$\frac{1}{4} - \frac{5}{4} = \frac{-4}{4} = -1$

with Positive index

$a^{1/4} \times a^{-5/4}$

$= a^{-1}$

$= \left(\frac{1}{a}\right)^1$

11. By using the substitution $y = 3^x$, find the two values of x such that (3)$3^x + 3^{-x} = 4$ and verify each solution by substitution into the original exponential equation.

(7.7)

$y = 3^x$

$\Rightarrow 3^{-x} = \frac{1}{y}$

$\times y$

Solve

$(3)3^x + 3^{-x} = 4$

$\Rightarrow 3y + \frac{1}{y} = 4$

$\Rightarrow 3y^2 + 1 = 4y$

$3y^2 - 4y + 1 = 0$

$(3y - 1)(y - 1) = 0$

$y = \frac{1}{3}, \quad y = 1$

Values of x?

$y = \frac{1}{3} = 3^x = 3^{-1} \Rightarrow x = -1$

$y = 1 = 3^x = 3^0 \Rightarrow x = 0$

Verify

$x=-1$: $(3)3^{-1} + 3^{--1} = 3\left(\frac{1}{3}\right) + 3 = 4$ ✓

$x=0$: $(3)3^0 + 3^{-0} = 3(1) + 1 = 4$ ✓

(7.7) **14.** By letting $2^x = y$, solve the equation $2^{x+1} + 2(2^{-x}) - 5 = 0$.

note: $2^{x+1} = (2)2^x$

If $2^x = y$

$2^{-x} = \frac{1}{y}$

$\times y$

solve

equation is $(2)2^x + 2(2^{-x}) - 5 = 0$

$\Rightarrow 2y + 2\left(\frac{1}{y}\right) - 5 = 0$

$\Rightarrow$

$2y^2 + 2 - 5y = 0$

$2y^2 - 5y + 2 = 0$

$(2y-1)(y-2) = 0$

$y = 1/2 \; , \; y = 2$

$x = ?$

$y = \frac{1}{2} = 2^{-1} = 2^x \Rightarrow x = -1$

$y = 2 = 2^1 = 2^x \Rightarrow x = 1$

6. Prove that for all real values of x,

(7.5) (i) $x^2 + 6x + 9 \geqslant 0$ (ii) $x^2 - 10x + 25 \geqslant 0$ (iii) $x^2 + 4x + 6 \geqslant 0$

(iv) $x^2 - 6x + 10 \geqslant 0$ (v) $4x^2 + 12x + 11 \geqslant 0$ (vi) $4x^2 - 4x + 2 \geqslant 0$.

This is a perfect square (i)

factorise

$x^2 + 6x + 9 \geq 0$

$(x+3)(x+3) \geq 0$

$(x+3)^2 \geq 0$ ✓ true

This is a perfect square (ii)

factorise

$x^2 - 10x + 25 \geq 0$

$(x-5)(x-5) \geq 0$

$(x-5)^2 \geq 0$ ✓ true

6. Prove that for all real values of x,

(7.5)

(i) $x^2 + 6x + 9 \geqslant 0$ (ii) $x^2 - 10x + 25 \geqslant 0$ (iii) $x^2 + 4x + 6 \geqslant 0$

(iv) $x^2 - 6x + 10 \geqslant 0$ (v) $4x^2 + 12x + 11 \geqslant 0$ (vi) $4x^2 - 4x + 2 \geqslant 0$.

This is not a perfect square

(iii) $x^2 + 4x + 6 \geq 0$

write in complete square form

Find a related complete square

$\frac{4x}{2} = 2x$

	x	$+2$
x	x^2	$2x$
$+2$	$2x$	$+4$

$\Rightarrow (x+2)^2$

$= x^2 + 4x + 4$

$\Rightarrow x^2 + 4x + 6$

$= x^2 + 4x + 4 + 2$

$= (x+2)^2 + 2$

$\Rightarrow x^2 + 4x + 6 \geq 0$

note its value is always > 0

since $(x+2)^2 + 2 \geq 0$

6. Prove that for all real values of x,

(7.5)

(i) $x^2 + 6x + 9 \geqslant 0$ (ii) $x^2 - 10x + 25 \geqslant 0$ (iii) $x^2 + 4x + 6 \geqslant 0$

(iv) $x^2 - 6x + 10 \geqslant 0$ (v) $4x^2 + 12x + 11 \geqslant 0$ (vi) $4x^2 - 4x + 2 \geqslant 0$.

This is not a perfect square

(iv) $x^2 - 6x + 10 \geq 0$

write in complete square form

Find a related complete square

$\frac{-6x}{2} = -3x$

	x	-3
x	x^2	$-3x$
-3	$-3x$	$+9$

$(x-3)^2$

$= x^2 - 6x + 9$

$\Rightarrow x^2 - 6x + 10$

$= x^2 - 6x + 9 + 1$

$= (x-3)^2 + 1$

$\Rightarrow x^2 - 6x + 10 \geq 0$

note its value is always > 0

since $(x-3)^2 + 1 \geq 0$

6. Prove that for all real values of x,

(7.5) (i) $x^2 + 6x + 9 \geqslant 0$ (ii) $x^2 - 10x + 25 \geqslant 0$ (iii) $x^2 + 4x + 6 \geqslant 0$
(iv) $x^2 - 6x + 10 \geqslant 0$ (v) $4x^2 + 12x + 11 \geqslant 0$ (vi) $4x^2 - 4x + 2 \geqslant 0.$

This is not a perfect square (v)

$4x^2 + 12x + 11 \geq 0$

$\div 4$

$x^2 + 3x + 11/4 \geq 0$

write in complete square form

Find a related complete square

$\frac{3x}{2} = \frac{3}{2}x$

	x	$+3/2$
x	x^2	$\frac{3}{2}x$
$+\frac{3}{2}$	$\frac{3}{2}x$	$+\frac{9}{4}$

$(x + 3/2)^2$
$= x^2 + 3x + 9/4$

note: $\frac{2}{4} = \frac{1}{2}$

$\Rightarrow x^2 + 3x + 11/4$
$= x^2 + 3x + 9/4 + 2/4$

$\Rightarrow x^2 + 3x + 11/4 \geq 0$

note its value is always > 0

Since $(x + 3/2)^2 + \frac{1}{2} \geq 0$

6. Prove that for all real values of x,

(7.5) (i) $x^2 + 6x + 9 \geqslant 0$ (ii) $x^2 - 10x + 25 \geqslant 0$ (iii) $x^2 + 4x + 6 \geqslant 0$
(iv) $x^2 - 6x + 10 \geqslant 0$ (v) $4x^2 + 12x + 11 \geqslant 0$ (vi) $4x^2 - 4x + 2 \geqslant 0.$

This is not a perfect square

$4x^2 - 4x + 2 \geq 0$

$\div 4$

$x^2 - x + 1/2 \geq 0$

write in complete square form

Find a related complete square

$\frac{-x}{2} = -\frac{1}{2}x$

	x	$-1/2$
x	x^2	$-\frac{1}{2}x$
$-\frac{1}{2}$	$-\frac{1}{2}x$	$+\frac{1}{4}$

$(x - \frac{1}{2})^2$
$= x^2 - x + 1/4$

$\Rightarrow x^2 - x + \frac{1}{2}$
$= x^2 - x + 1/4 + 1/4$

$\Rightarrow x^2 - x + \frac{1}{2} \geq 0$

note its value is always > 0

Since $(x - 1/2)^2 + 1/4 \geq 0$