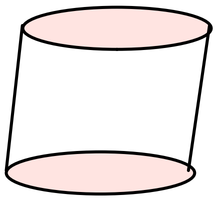
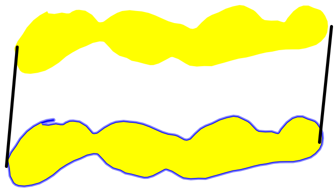
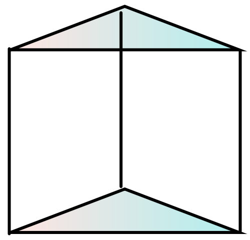
Area and Volume

Scetion 6.3







Prism Volume = Ah

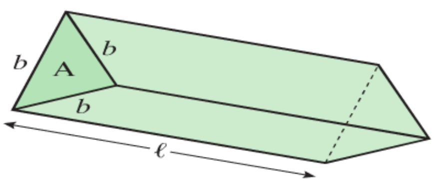
Section 6.3 3-Dimensional objects

1. Prisms

A **prism** is a three-dimensional figure that has the same cross-section all along its length. Shown here is a standard triangular prism.

The volume = $(A \times l)\ \text{m}^3$

The external surface area = $[2A + 3(l \times b)]\ \text{m}^2$



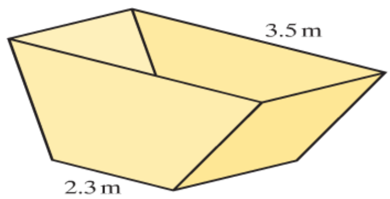
A rubbish skip is also a prism where the base is in the shape of a trapezium.

Given that the width and perpendicular height of this skip are both 1.8 m,

the volume $V = \text{Area}_{\text{trapezium}} \times 1.8\ \text{m}^3$

$$= \left(\frac{3.5 + 2.3}{2}\right) \times 1.8 \times 1.8\ \text{m}^3$$

$$= 9.369\ \text{m}^3 = 9.4\ \text{m}^3.$$



The surface area of a prism is best obtained by expanding the net of the prism.

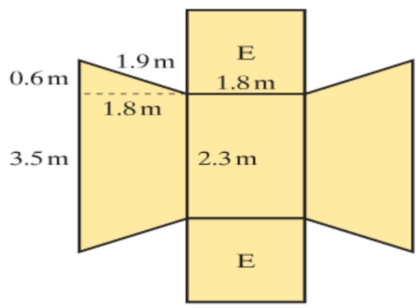
The slant height is $\sqrt{0.6^2 + 1.8^2} = 1.9\ \text{m}$

The area of each end (E) = $(1.8 \times 1.9)\ \text{m}^2$

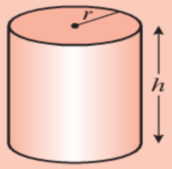
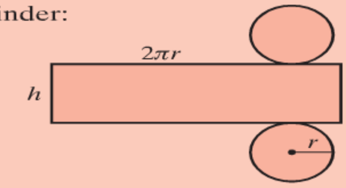
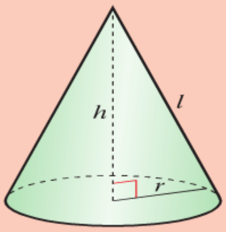
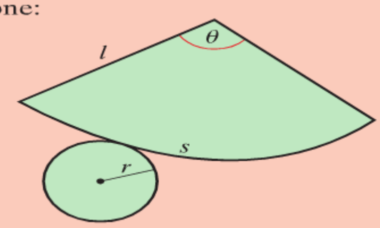
This skip has an external surface area of:

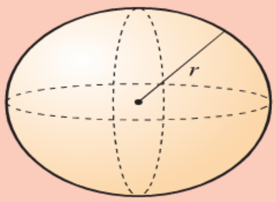
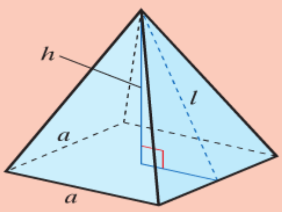
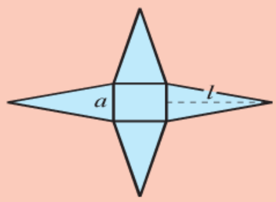
$$\text{Area} = 2\left(\frac{3.5 + 2.3}{2}\right) \times 1.8 + 1.8 \times 2.3 + 2(1.8 \times 1.9)$$

$$= 21.42\ \text{m}^2$$



2. Revision of Cylinder, Cone and Sphere

<table>
<tr><th>Shape</th><th>Diagram</th><th>Properties</th></tr>
<tr><td>Cylinder</td><td>
</td><td>Volume $= \pi r^2 \times h$
Surface Area $= 2 \times \pi r^2 + 2\pi r \times h$
Net of cylinder:

</td></tr>
<tr><td>Cone</td><td>
</td><td>Volume $= \frac{1}{3}\pi r^2 \times h$
Surface Area $= \pi r^2 + \pi rl$
Net of cone:


Note: (i) In a right circular cone, the apex is directly over the centre of the base. Hence, $l^2 = r^2 + h^2$.
(ii) The arc length of the sector (s) $=$ circumference of the base $(2\pi r)$.</td></tr>
<tr><td>Sphere</td><td>
</td><td>$\text{Volume}_{\text{sphere}} = \frac{4}{3}\pi r^3$
$\text{Volume}_{\text{hemisphere}} = \frac{2}{3}\pi r^3$
Surface area of sphere $= 4\pi r^2$
Surface area of hemisphere $= 3\pi r^2$</td></tr>
<tr><td>Pyramid</td><td>
</td><td>Volume $= \frac{1}{3}a^2 \times h$
Surface Area $= a^2 + 4(\frac{1}{2}al) = a^2 + 2al$
$l^2 = h^2 + \frac{a^2}{4}$
Net of pyramid:

</td></tr>
</table>

Note: Both a prism and a pyramid can have many differently-shaped (polygonal) bases.

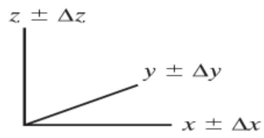
Generally, (i) the volume of a **prism** $=$ (area of the base) $\times h$

(ii) the volume of a **pyramid** $= \frac{1}{3}$(area of the base) $\times h$

3. Degrees of accuracy

When a measurement is made to a given degree of accuracy, an *error* on the measurement is created. If x is the measurement, the real measurement could be in the range $x \pm \Delta x$.

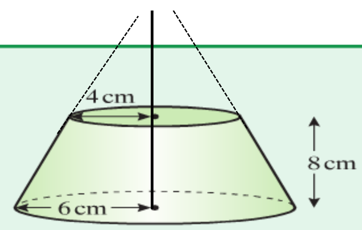
E.g. If a length is measured as 10.3 cm, corrected to one place of decimals, this implies a minimum possible length of 10.25 cm and a maximum possible length of 10.34 cm.



If the measurement is area or volume, each dimension will have a similar range.

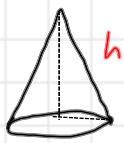
Example 1

Find the volume of the truncated cone shown (a frustum) correct to 1 place of decimals.



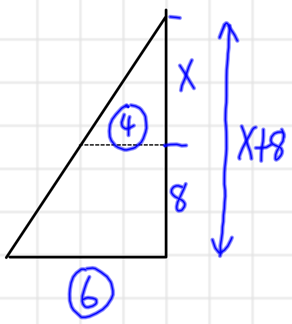
$V_{cone} = \frac{1}{3}\pi R^2 h$

large cone



$h = 16 + 8 = 24$

$R = 6$



Scale factor $\frac{3}{2}$

$x\left(\frac{3}{2}\right) = x + 8$

$3x = 2x + 16$

$x = 16$

Example 2

A company makes ball bearings (spheres) for a machine with a diameter of 12 mm. They claim that they are produced to an accuracy of ±0.02 mm. Find the largest and smallest ball bearing volumes produced.

Find the percentage error on (i) the diameter (ii) the volume.

$V = \frac{4}{3}\pi R^3$

Diameter large = 12.02 mm

R_L = 6.01 m

Diameter Small = 11.98 mm

R_S = 5.99 mm

% ERROR

$= \frac{\text{ERROR}}{\text{"right"}} \times 100$

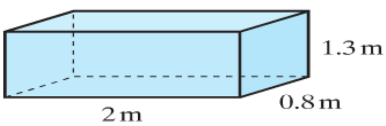
Exercise 6.3

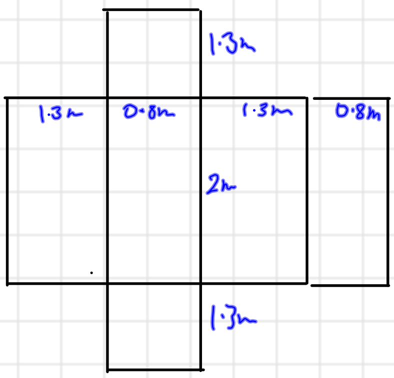
1. Examine each of the following shapes closely and

(i) by drawing a suitable net of each, calculate the total area correct to one place of decimals

(ii) find the volume of each shape correct to one place of decimals.

(a)





(i) Net

Area = 2(LB + BH + LH)

$A = 2\left((1.3)(0.8) + (2)(1.3) + (2)(0.8)\right)$

$= 6.28 \approx 6.3\ m^2$ (1 d.p.)

$V = LBH$

$V = (1.3)(2)(0.8)$

$= 2.08 \approx 2.1\ m^3$ (1 dp)

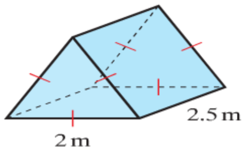
Exercise 6.3

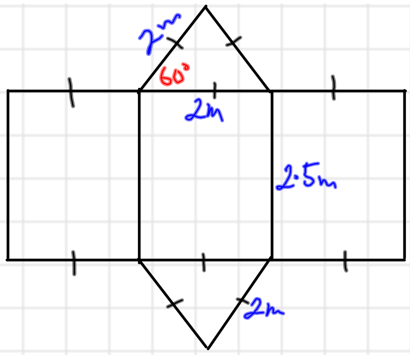
1. Examine each of the following shapes closely and

(i) by drawing a suitable net of each, calculate the total area correct to one place of decimals

(ii) find the volume of each shape correct to one place of decimals.

(b)





$A = LB$

$\Delta = \frac{1}{2}ab\sin C$

Area = 2 rectangles + 2 triangles

$= (2)(2.5) + 2\left(\frac{1}{2}(2)(2)\sin 60°\right)$

$= 8.46 \approx 8.5\ m^2$

Volume of Prism $= Ah$

Volume = Area of Δ × height

$= \frac{1}{2}(2)(2)\sin 60° \times 2.5$

$= 4.33 \approx 4.3\ m^3$

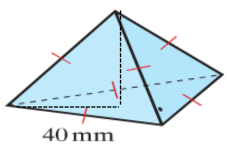
Exercise 6.3

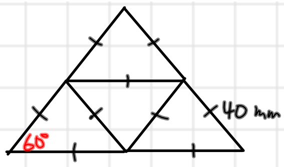
1. Examine each of the following shapes closely and

(i) by drawing a suitable net of each, calculate the total area correct to one place of decimals

(ii) find the volume of each shape correct to one place of decimals.

(c)





$\Delta = \frac{1}{2}ab\sin C$

Area = 4 triangles

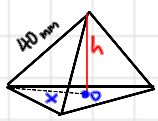
$= 4\left(\frac{1}{2}\right)(40)(40)\sin 60°$

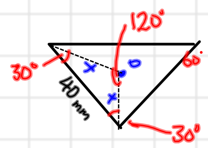
$= 2771.28 \approx 2{,}771.3\ mm^2$

Tetrahedron Volume $= \frac{1}{3}Ah$

Area Base

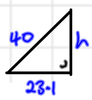
height ?





$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 30°} = \frac{40}{\sin 120°}$, $x = \frac{40\sin 30°}{\sin 120°} \approx 23.1\ mm$



$h^2 = 40^2 - 23.1^2 \Rightarrow h \approx 32.7\ mm$

Volume ?

$V = \frac{1}{3}\left(\frac{1}{2}(40)(40)\sin 60\right)(32.7) \simeq 7{,}541.5\ mm^3$

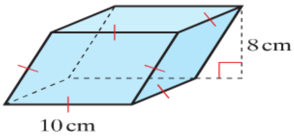
Exercise 6.3

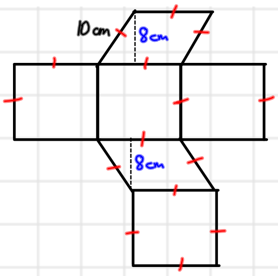
1. Examine each of the following shapes closely and

(i) by drawing a suitable net of each, calculate the total area correct to one place of decimals

(ii) find the volume of each shape correct to one place of decimals.

(d)





Area = 4 Squares + 2 parallelograms
= $4(10)^2 + 2(10)(8)$
= $560\ cm^2$

Prism Volume = Ah

Volume = Area parallelogram × height
= $(10)(8) \times 10$
= $800\ cm^3$

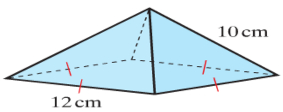
Exercise 6.3

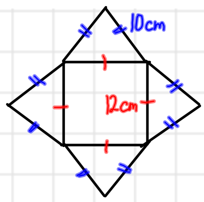
1. Examine each of the following shapes closely and

(i) by drawing a suitable net of each, calculate the total area correct to one place of decimals

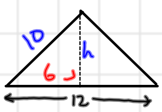
(ii) find the volume of each shape correct to one place of decimals.

(e)





Area = 4 triangles + Square



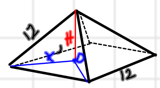
$h^2 = 10^2 - 6^2 = 64$
$h = 8\ cm$

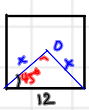
$A = \frac{Bh}{2}$

Area = $4\left(\frac{(6)(8)}{2}\right) + (12)^2 = 240\ cm^2$

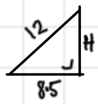
$V = \frac{1}{3}Ah$
Base Area

height of pyramid? H





$\sin 45° = \frac{x}{12}$
$x = 12\sin 45° = 8.5cm$



$H^2 = 12^2 - 8.5^2$
$H \simeq 8.5\ cm$

$V = \frac{1}{3}(12^2)(8.5) = 408\ cm^3$

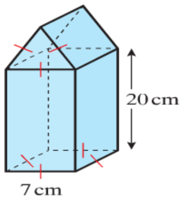
Exercise 6.3

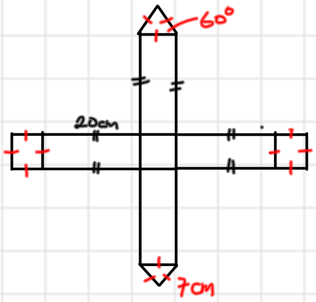
1. Examine each of the following shapes closely and

(i) by drawing a suitable net of each, calculate the total area correct to one place of decimals

(ii) find the volume of each shape correct to one place of decimals.

(f)





Area
= 3 squares
+ 4 rectangles
+ 2 triangles

$\Delta = \frac{1}{2}ab\sin C$

$$\text{Area} = 3(7)^2 + 4(20)(7) + \tfrac{1}{2}(7)(7)\sin 60^\circ$$

$$\approx 728.2 \text{ cm}^2$$

Prism Volume $= Ah$

V = Area Side with rectangle and triangle × width

$$= \left[(20)(7) + \tfrac{1}{2}(7)(7)\sin 60^\circ\right] \times 7$$

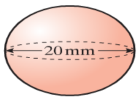
$$\approx 1128.5 \text{ cm}^3$$

2. In a woodwork class, the students were asked to list in order from largest to smallest, the (i) volume (ii) **total** surface area of each of the following 3-dimensional objects, each answer given correct to the nearest whole number.
Make two separate lists for (i) the areas (ii) the volumes, each arranged in descending order.

(i) Volumes?

nearest whole number

(a)

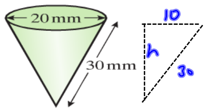


$V = \frac{4}{3}\pi R^3$

$= \frac{4}{3}(3.14)(10)^3$

$\approx 4187 \text{ mm}^3$

(b)



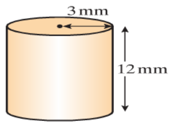
$h = \sqrt{30^2 - 10^2} = 20\sqrt{2}$

$V = \frac{1}{3}\pi R^2 h$

$= \frac{1}{3}(3.14)(10)^2(20\sqrt{2})$

$\approx 2960 \text{ mm}^3$

(c)

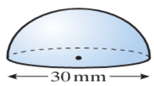


$V = \pi R^2 h$

$= (3.14)(3^2)(12)$

$\approx 339 \text{ mm}^3$

(d)

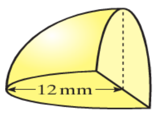


$V = \frac{2}{3}\pi R^3$

$= \frac{2}{3}(3.14)(15)^3$

$\approx 7065 \text{ mm}^3$

(e)

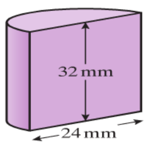


$V = \frac{1}{3}\pi R^3$

$= \frac{1}{3}(3.14)(12)^3$

$\approx 1809 \text{ mm}^3$

(f)

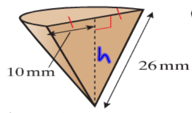


$V = \frac{1}{2}\pi R^2 h$

$= \frac{1}{2}(3.14)(12)^2(32)$

$\approx 7235 \text{ mm}^3$

(g)



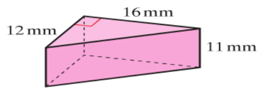
$h = \sqrt{26^2 - 10^2} = 24$

$V = \frac{1}{3}\pi R^2 h$

$= \frac{1}{6}(3.14)(10)^2(24)$

$\approx 1256 \text{ mm}^3$

(h)

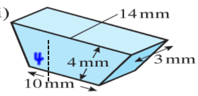


$V = Ah$

$= \left[\frac{1}{2}(12)(16)\right](11)$

$= 1056 \text{ mm}^3$

(i)



$V = Ah$

$= \left[\left(\frac{14+10}{2}\right)4\right](3)$

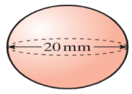
$= 144 \text{ mm}^3$

2. In a woodwork class, the students were asked to list in order from largest to smallest, the (i) volume (ii) **total** surface area of each of the following 3-dimensional objects, each answer given correct to the nearest whole number.
Make two separate lists for (i) the areas (ii) the volumes, each arranged in descending order.

(ii) Total Surface Area?

nearest whole number

(a)

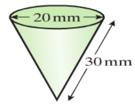


$TSA = 4\pi R^2$

$= 4(3.14)(10)^2$

$\approx 1256 \text{ mm}^2$

(b)

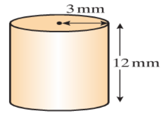


$TSA = \pi R \ell + \pi R^2$

$= (3.14)(10)(30) + (3.14)(10)^2$

$\approx 1256 \text{ mm}^2$

(c)



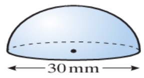
$TSA = 2\pi R h + 2\pi R^2$

$= 2\pi R(h+R)$

$= 2(3.14)(3)(12+3)$

$\approx 283 \text{ mm}^2$

(d)

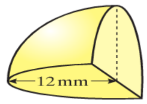


$TSA = \frac{4\pi R^2}{2} + \pi R^2$

$= 3\pi R^2 = 3(3.14)(15)^2$

$\approx 2120 \text{ mm}^2$

(e)

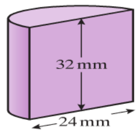


$TSA = \frac{4\pi R^2}{2} + 2\left[\frac{\pi R^2}{2}\right]$

$= 3\pi R^2 = 3(3.14)(12)^2$

$\approx 1356 \text{ mm}^2$

(f)



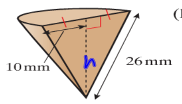
$TSA = 2\left[\frac{\pi R^2}{2}\right] + LB + \frac{2\pi r h}{2}$

$= \pi R^2 + 2Rh + \pi R h$

$= 3.14(12^2) + 2(12)(32) + (3.14)(12)(32)$

$\approx 2426 \text{ mm}^2$

(g)



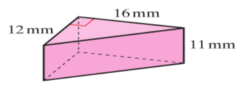
$h = \sqrt{26^2 + 10^2} = 24$

$TSA = \frac{Bh}{2} + \frac{\pi R L}{2} + \frac{\pi R^2}{2}$

$= \frac{(20)(24)}{2} + \frac{(3.14)(10)(26)}{2} + \frac{(3.14)(10^2)}{2}$

$\approx 805 \text{ mm}^2$

(h)

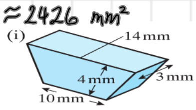


$TSA = LB + BH + LH + 2\left[\frac{BL}{2}\right]$

$= (12)(11) + (16)(12) + (11)(16) + (12)(16)$

$= 692 \text{ mm}^2$

(i)



TSA = Area of 6 sides

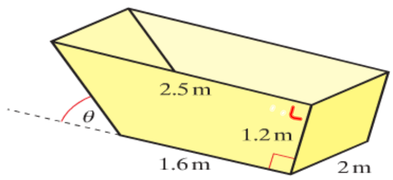
$= 2(3)(4) + 14(3) + 10(3) + 2\left[\frac{(14+10)}{2}(4)\right]$

$= 144 \text{ mm}^2$

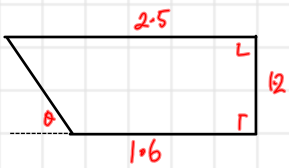
3. This model of a skip is used by a recycling company.

(i) Find the volume of the skip, correct to two places of decimals.

(ii) The company offers a 'volume pick-up' at €80 per m³ or a 'weight pick-up' at €30 per 100 kg, assuming a full skip weighs 1.3 tonnes. Which option represents the best "value for money" for the customer?



(i) Prism



$V = Ah$

$A = \left(\frac{a+b}{2}\right)h = \frac{(2.5)(1.6)}{2}(1.2) = 2.46$

$V = Ah = 2.46(2) = 4.92 \text{ m}^3$

(ii) option 1

Cost of €80 per m³ option?

Cost $= 80(4.92) =$ €393.60

option 2

1 tonne = 1000 kg

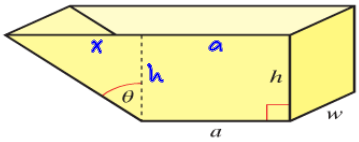
1.3 tonnes = 1.3 × 1000 kg = 1,300 kg

= 13 × 100 kg

If 100 kg costs €30

→ cost = 13(30) = €390.00 ✓ BEST VALUE

(iii) Write an equation for the volume of the skip in terms of a, h, w and θ.



$V = Ah$

$A = \frac{(a+b)}{2}h$

$V = \text{Area of trapezium} \times w$

$x = ?$ $\quad \tan\theta = x/h$

$\Rightarrow x = h\tan\theta$

$$V = \left[\frac{(a + a + h\tan\theta)h}{2}\right] w$$

$$V = \frac{hw}{2}\left(a^2 + h\tan\theta\right)$$

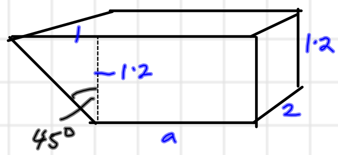
(iv) The recycling company wants to redesign the skip with a new angle $\theta = 45^o$. If the width, height and overall volume must remain the same in order to fit on the truck, find, correct to one place of decimals, the new dimensions of the top and the bottom of the skip.

$w = 2m$

$h = 1{\cdot}2m$

$V = 4{\cdot}92\ m^3$

$\theta = 45°$



$V = \frac{hw}{2}\left(a^2 + h\tan\theta\right)$

$$4{\cdot}92 = \frac{(1{\cdot}2)(2)}{2}\left[a^2 + 2\tan 45°\right]$$

$$4{\cdot}92 = 1{\cdot}2\left[a^2 + 2\right]$$

$\div 1{\cdot}2$ $\quad 4{\cdot}1 = a^2 + 2$

-2 $\quad 2{\cdot}1 = a^2$

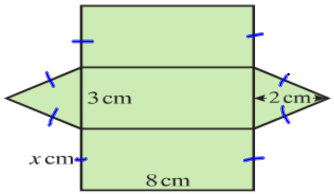
$a = \sqrt{2{\cdot}1} = 1{\cdot}449 \approx 1{\cdot}4\ m$

Bottom $\quad a = 1{\cdot}4\ m$

Top $\quad a + 1{\cdot}2 = 2{\cdot}6\ m$

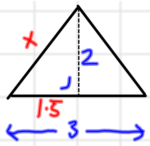
4. The net of a 3D figure is shown in the diagram. Both triangles are isosceles and congruent.

(i) Calculate the length of the side x cm.
(ii) Draw a sketch of the 3D figure and name it.
(iii) Calculate its volume.
(iv) Design a trapezoidal prism with the same volume.



x?

Pythagoras

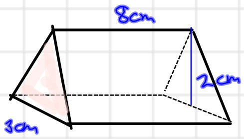


$x^2 = 2^2 + 1.5^2$

$x = \sqrt{2^2 + 1.5^2} = 2.5$ cm

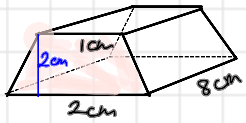
3D Sketch

Its a Prism



$V = Ah$

$V = \left[\frac{(2)(3)}{2}\right]8 = 24$ cm^3

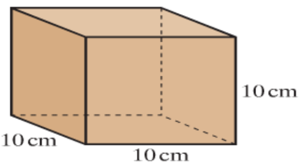


$V = \left[\frac{(1+2)(2)}{2}\right]8$

$= 24$ cm^3

5. (i) A student in a woodwork class is asked to fashion the largest sphere possible from the cube opposite. What volume of wood must be chipped away?

(ii) The student is then asked to calculate the volume of the smallest sphere that can enclose the cube fully.



$V = LBH$

$V = 10^3 = 1000$ cm^3

$V_{sphere} = \frac{4}{3}\pi R^3$

$R = 5$ cm

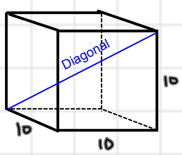
$V = \frac{4}{3}(3.14)(5)^3$

≈ 523.3 cm^3

Volume chipped away = Volume Cube - Volume Sphere

$V = 1000 - 523.3 \approx 476.7$ cm^3

Smallest enclosing sphere?



Diameter of enclosing sphere = diagonal of cube.

$D^2 = 10^2 + 10^2 + 10^2$

$D^2 = 3000$

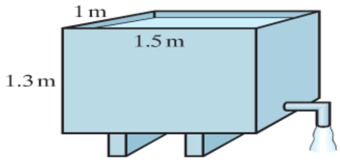
$D = \sqrt{3000} = 10\sqrt{30}$

Radius, $R = 5\sqrt{30}$

$V = \frac{4}{3}\pi R^3$

$V = \frac{4}{3}(3.14)(5\sqrt{30})^3 = 3439.7$ cm^3

6. A water tank, in the shape of a cuboid(rectangular solid), is full of water. Water is drained from the tank at a rate of 8 litres per minute.
The dimensions of the tank are given to the nearest 10 cm.
The rate at which the water is drained from the tank is given to the nearest 0.5 litres per minute.
Calculate, correct to the nearest minute,
(i) the shortest time possible to drain the tank
(ii) the longest time possible to drain the tank.

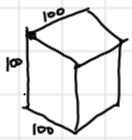


Full tank
$V = LBh$

$V_{tank} = (1)(1\cdot3)(1\cdot5) = 1\cdot95 \ m^3$

Litres?

1 litre = 1000 cm^3 (mL)



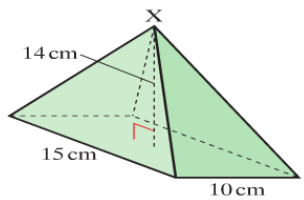
$1 m^3 = 1{,}000{,}000 \ cm^3$
$1 m^3 \quad 1{,}000$ litres
$\Rightarrow 1\cdot95 m^3 = 1{,}950$ litres

* note: to the nearest 0·5 litres means ±0·25 litres

shortest time? Rate of flow = $8 + 0\cdot25 = 8\cdot25$ l/min.
Time = $1950 / 8\cdot25 \approx 236$ minutes

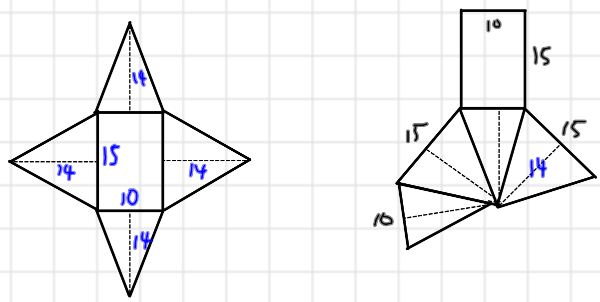
longest time? Rate of flow = $8 - 0\cdot25 = 7\cdot75$ l/min
Time = $1950 / 7\cdot75 = 252$ minutes

7. The pyramid shown opposite has a rectangular base.
The point X is directly above the midpoint of the base.
(i) Find the volume of the pyramid.
(ii) Draw two different possible nets of the pyramid and hence (using one of them) find the total external surface area of the pyramid.



$V = \frac{1}{3}Ah$

$V = \frac{1}{3}[(10)(15)]\,14 = 700 \ cm^3$



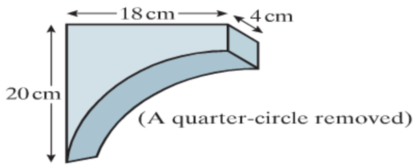
$\Delta = \frac{Bh}{2}$

Surface area = 4 triangles + rectangle

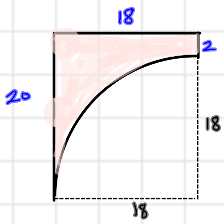
$= 2\left[\frac{(14)(15)}{2}\right] + 2\left[\frac{(10)(14)}{2}\right] + (10)(15)$

$= 500 \ cm^2$

8. A steel support is to be made from a rectangular block of metal 4 cm thick, as shown.
If a quarter-circle is removed, calculate the total surface area and the total volume of the support.



Area



$A = LB - \frac{1}{4}\pi R^2$

$= (20)(18) - \frac{1}{4}(3.14)(18)^2$

$\approx 105.7 \text{ cm}^2$

Volume prism = Ah

$V = (105.7)(4) \approx 422.6 \text{ cm}^3$

9. If x, y, and z represent lengths, and π and a are numbers with no dimensions, state whether each of the following formulae represents

(a) $\pi x^2 + \pi y^2 + \pi z^2$

Area
(2-dimensional)

(b) $ax + \pi y$

Length
(1-dimensional)

(c) axz

Area
(2-dimensional)

(d) $a\pi y$

Length
(1-dimensional)

(e) $axy + \pi az$

Area
(2-dimensional)

(f) $ax + xy$

Area
(2-dimensional)

(g) $axyz$

Volume
(3-dimensional)

(h) $x^2y + y^2z + z^2x$

Volume
(3-dimensional)

10. If A represents area, V represents volume, and x, y, z are lengths, which of these formulae are consistent, and which are inconsistent, in terms of dimensions? Explain your answers.

(i) $Ax = z^3$

consistent

3-dimensional = 3-dimensional

(ii) $x = \frac{V}{Ay}$

inconsistent

1-dimensional ≠ 0-dimensional

(iii) $V = xy + z$

inconsistent

3-dimensional ≠ 2-dimensional

(iv) $A = x^2 + y^2 + z^2$

consistent

2-dimensional = 2-dimensional

(v) $V = A(x + y + z)$

consistent

3-dimensional = 3-dimensional

(vi) $A = \frac{V}{x} + y$

consistent

2-dimensional = 2-dimensional

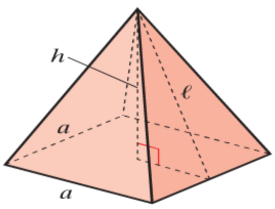
(vii) $x = y + z$

consistent

1-dimensional = 1-dimensional

11. The formula for the volume of a pyramid is
$V = \frac{1}{3}$(base area) × perpendicular height.

(i) For the square-based pyramid opposite, find the volume in terms of a and h.
Find the volume of a pyramid if the length of the base is 6 cm and the height is 7 cm.



$V = \frac{1}{3}$ (base area) × perpendicular height

$$V = \frac{1}{3} a^2 h$$

$a = 6$ cm
$h = 7$ cm

$$V = \frac{1}{3}(6^2)(7) = 84 \text{ cm}^3$$

(ii) Another square-based pyramid has a base length of 5 cm and a volume of 100 cm^3.
Find its perpendicular height.
Hence, and by drawing its net, find the total surface area.

$V = \frac{1}{3}a^2h$

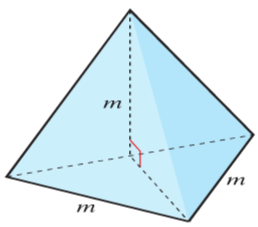
$a = 5$ cm
$h = ?$
$V = 100\ cm^3$

$\Rightarrow \quad 100 = \frac{1}{3}(5)^2 h$

$100 = \frac{25}{3} h$

$h = \frac{3(100)}{25} = 12$ cm

(iii) The pyramid shown opposite has been cut from a square-based pyramid. Find its volume in terms of m. Describe this pyramid. Draw its net and find its total surface area in terms of m.

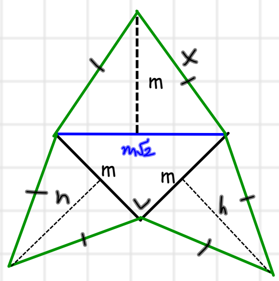


Full pyramid
$V = \frac{1}{3}a^2h$

This shape has 1/2 the volume of a square based pyramid with:
height = m and Base = m

$\Rightarrow \quad V = \frac{1}{2}\left[\frac{1}{3}m^2 m\right] = \frac{m^3}{6}$

Net



Pytagoras

$x^2 = m^2 + \left(\frac{m\sqrt{2}}{2}\right)^2$

$x^2 = m^2 + m^2$
$x^2 = 2m^2$
$x = m\sqrt{2}$

Area = 4 triangles
$\Delta = \frac{Bh}{2}$

$h^2 = (m\sqrt{2})^2 - \left(\frac{m}{2}\right)^2 = 2m^2 - \frac{m^2}{4} = \frac{7}{4}m^2 \Rightarrow h = \frac{m\sqrt{7}}{2}$

$Area = \frac{1}{2}(m)(m) + 2\left[\frac{1}{2}(m\sqrt{7})(m)\right] + \frac{1}{2}(m\sqrt{2})(m)$

$= \frac{1}{2}\left[m^2 + 2\sqrt{7}\,m^2 + \sqrt{2}\,m^2\right] \approx 3.85\ m^2$

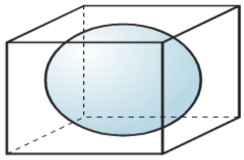
12. (i) A solid sphere just fits into a cubical box, as shown. If the edge of the box is 14 cm in length, and taking $\pi = \frac{22}{7}$, find

(a) the volume of the box in cm^3

(b) the volume of the sphere in cm^3

(c) the percentage of space not occupied by the sphere.

Give your answer correct to the nearest integer.



14 cm

$V = LBH$

(a) $V = 14^3 = 2744 \text{ cm}^3$

$V = \frac{4}{3}\pi R^3$

$R = 7 \text{ cm}$

(b) $V = \frac{4}{3}\left(\frac{22}{7}\right)(7)^3 = 1437\frac{1}{3} \text{ cm}^3$

(c) % free space?

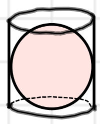
Amount free space $= V_{BOX} - V_{SPHERE}$

$V_{FREE} = 2744 - 1437\frac{1}{3} = 1306\frac{2}{3} \text{ cm}^3$

$\% \text{ Free Space} = \dfrac{1306\frac{2}{3} \times 100}{2744} \approx 48\%$

(ii) The same solid sphere fits exactly inside a cylinder.
Determine if the percentage of unoccupied space in the cylinder is greater or less than the space unoccupied in the cubical box.

Sketch



14cm

$R = 7 \text{ cm}$

$V_{cylinder} = \pi R^2 h$

$V_{sphere} = \frac{4}{3}\pi R^3$

$V_{cylinder} = \left(\frac{22}{7}\right)(7)^2(14) = 2156 \text{ cm}^3$

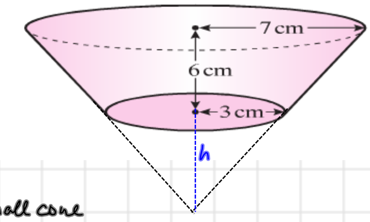
$V_{sphere} = \frac{4}{3}\left(\frac{22}{7}\right)(7)^3 = 1437\frac{1}{3} \text{ cm}^3$

$V_{free\ space} = 2156 - 1437\frac{1}{3} = 718\frac{2}{3}$

$\% \text{ free space} = \dfrac{718\frac{2}{3} \times 100}{2156} \approx 33\%$

$\Rightarrow$ this is less than 48% unoccupied in cube.

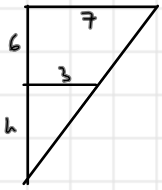
13. Find, correct to 1 place of decimals, the volume of this rubber stopper.



$V_{Stopper} = V_{Large\ cone} - V_{Small\ cone}$

$h = ?$

similar triangles



$$\frac{7}{3} = \frac{6+h}{h}$$

$$\Rightarrow 7h = 3(6+h)$$

$$7h = 18 + 3h$$

$$4h = 18$$

$$h = 9/2$$

$V_{cone} = \frac{1}{3}\pi R^2 h$

large cone $\begin{cases} h = 6 + \frac{9}{2} = \frac{21}{2} \\ R = 7 \end{cases}$

Small cone $\begin{cases} h = 9/2 \\ R = 3 \end{cases}$

$$V_{Large} = \frac{1}{3}\left(\frac{22}{7}\right)\left(7^2\right)\left(\frac{21}{2}\right) = 539 \text{ cm}^3$$

$$V_{small} = \frac{1}{3}\left(\frac{22}{7}\right)\left(3^2\right)\left(\frac{9}{2}\right) \approx 42.4 \text{ cm}^3$$

$$V_{Stopper} = 539 - 42.4 = 496.6 \text{ cm}^3$$