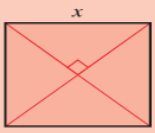
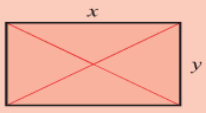
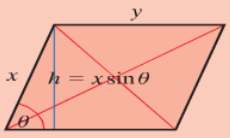
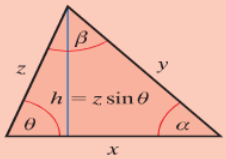
Area and Volume

Section 6.1

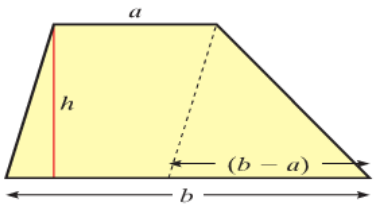
Key words

polygon **area** **perimeter** **diagonal** **trapezium** **cyclic quadrilateral**
arc **sector** **radian** **quadrilateral**

Shape	Diagram	Properties
Square		› all sides have the same length › all angles are 90° › perimeter $= 4x$ › area $= x^2$ › diagonal $= \sqrt{2}x$ › diagonals perpendicularly bisect each other
Rectangle		› opposite sides have the same length › all angles are 90° › perimeter $= 2(x + y)$ › area $= xy$ › diagonal $= \sqrt{x^2 + y^2}$ › diagonals have the same length › diagonals bisect each other
Parallelogram		› opposite sides have the same length › opposite angles are equal › perimeter $= 2(x + y)$ › area $= yh = yx\sin\theta$ › diagonals bisect each other
Triangle		› perimeter $= x + y + z$ › area $= \frac{1}{2}x.h = \frac{1}{2}x.z.\sin\theta$ › $\frac{y}{\sin\theta} = \frac{z}{\sin\alpha}$ › types include isosceles, equilateral, scalene, right-angled › $\alpha + \beta + \theta = 180°$ › special right-angled triangles with sides • $3, 4, \mathbf{5}$ ..... $(36.9°, 53.1°, 90°)$ • $1, \sqrt{3}, \mathbf{2}$, ... $(30°, 60°, 90°)$ • $1, 1, \sqrt{\mathbf{2}}$ ..... $(45°, 45°, 90°)$

1. Trapezium

A **trapezium** is a quadrilateral which has one pair of parallel sides.



The area of a trapezium $= ah_{\text{parallelogram}} + \frac{1}{2}(b-a)\,h_{\text{triangle}}$

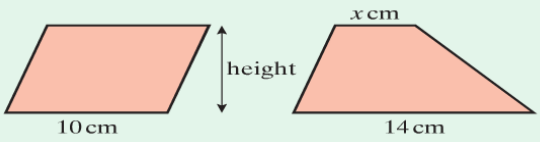
$= ah + \frac{1}{2}bh - \frac{1}{2}ah$

$= \frac{1}{2}ah + \frac{1}{2}bh = \left(\frac{a+b}{2}\right)h$

= half the sum of the lengths of the parallel sides times the height.

Example 1

If a parallelogram has a base of 10 cm, and a trapezium of the same area and height has a base of 14 cm, find x, the length of the other parallel side of the trapezium.



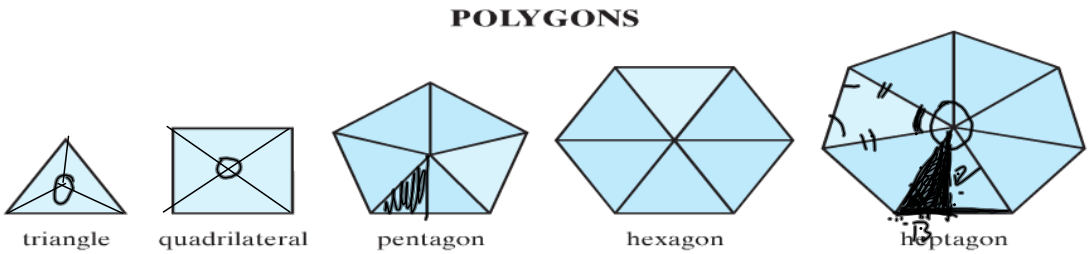
Area Par. = Area Trap.

$Bh = \left(\frac{a+b}{2}\right)h$

$10h = \left(\frac{x+14}{2}\right)h$

$20 = x + 14$

$6 = x$



2. Polygons

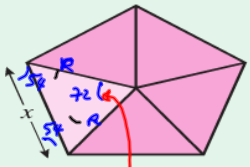
A **polygon** is a plane (2-dimensional) shape with straight edges.

Regular polygons are symmetrical, with a base triangle repeated in polygons with more than 4 sides. The interior angles of regular polygons are:

Triangle = 60°, Quadrilateral = 90°, Pentagon = 108°, Hexagon = 120°, Heptagon = 128.6°

Example 2

The area of the regular pentagon shown here is 600 cm^2. Calculate the length of one side, x, of the pentagon.



$\frac{360°}{5} = 72°$

Pentagon Area

1 Triangle Area

$A = 600$

$\Delta = \frac{600}{5} = 120 \text{ cm}^2$

Area Triangle

$\Delta = \frac{1}{2} ab \sin C$

$\frac{R^2}{2} \sin 72 = 120$

$R^2 = \frac{2(120)}{\sin 72} = 252.35$

$R = 15.9$

$\Delta = \frac{1}{2} ab \sin C$

$\frac{1}{2}(15.9) \times \sin 54 = 120$

$x = \frac{120(2)}{(15.9)\sin 54} = 18.7 \text{ cm}$