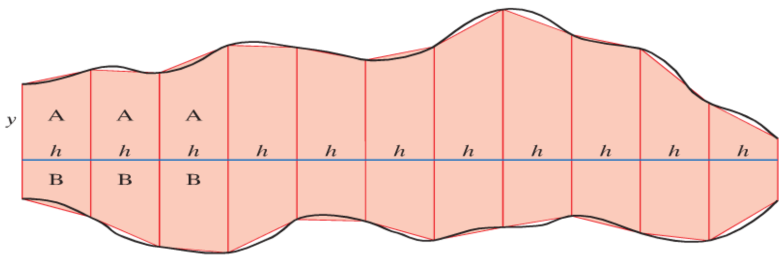
Area and Volume

Section 6.4

Section 6.4 Trapezoidal rule for calculating area

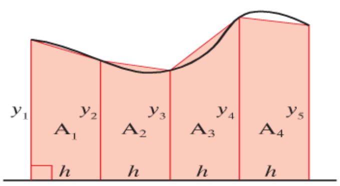
To calculate the areas of shapes with irregular boundaries, e.g. fields, lakes, etc., surveyors have usually divided the area into a series of parallel strips, each in the shape of a trapezium; a quadrilateral with two of the four sides parallel to each other.



A straight line is drawn across the centre of the area, dividing it into a series of two different areas, A and B.

The area of each section, above and below the line, can be calculated separately using the formula for the area of a trapezium and then added together.

Along the line and at equal intervals of h, perpendicular lines are drawn up to the boundary. These ordinates (offsets) – y_1, y_2, y_3, etc – are the parallel sides of the trapezium.



Using the area formula for a trapezium, $\frac{a+b}{2} \times h$, we get

$A_1 = \frac{y_1 + y_2}{2} \times h$. Similarly, $A_2 = \frac{y_2 + y_3}{2} \times h$, and so on.

Therefore, the total area $A = A_1 + A_2 + A_3 + A_4$.

$$= \left(\frac{y_1 + y_2}{2} \times h\right) + \left(\frac{y_2 + y_3}{2} \times h\right) + \left(\frac{y_3 + y_4}{2} \times h\right) + \left(\frac{y_4 + y_5}{2} \times h\right)$$

$$= \frac{h}{2}(y_1 + y_2 + y_2 + y_3 + y_3 + y_4 + y_4 + y_5)$$

$$= \frac{h}{2}[y_1 + 2(y_2 + y_3 + y_4) + y_5]$$

In words, Area $\approx \dfrac{\text{interval width}}{2}$ [first height + last height + 2(remaining heights)]

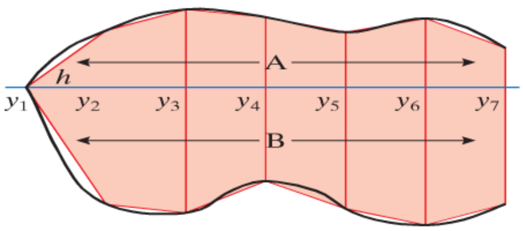
When n strips are made, the Trapezoidal formula becomes

$$\text{Area} \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + \ldots . y_{n-1})]$$

Note 1: Because the top of each trapezium does not match the boundary at all points, the area obtained by this formula is only approximate.
Its accuracy depends on the gap width h; the smaller the gap width, the greater the accuracy.

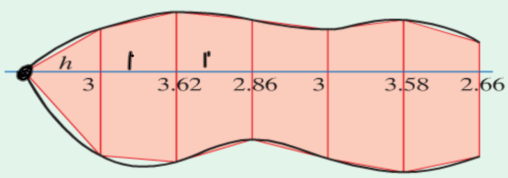
Note 2: If offsets are measured from the same points above and below the line, then the area (A + B) can be obtained using

$$\text{Area} \approx \frac{h}{2}[y_1 + y_7 + 2(y_2 + y_3 + y_4 + y_5 + y_6)]$$



Example 1

Using the measurements provided, find the area of this shape given $h = 1$ unit.



$h = 1$

$y_1 = 0$ $y_2 = 3$ $y_3 = 3.62$

$y_4 = 2.86$ $y_5 = 3$ $y_6 = 3.58$

$y_5 = 2.66$

Area $= \frac{1}{2}[0 + 2.66 + 2(3 + 3.62 + 2.86 + 3 + 3.58)]$

$= 17.3$