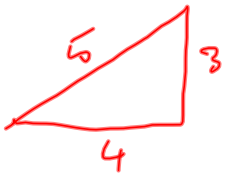
Circle Notes

from Class

Feb. 2012

P.229
Q24

centre? (0,0) ✓
Radius? 5



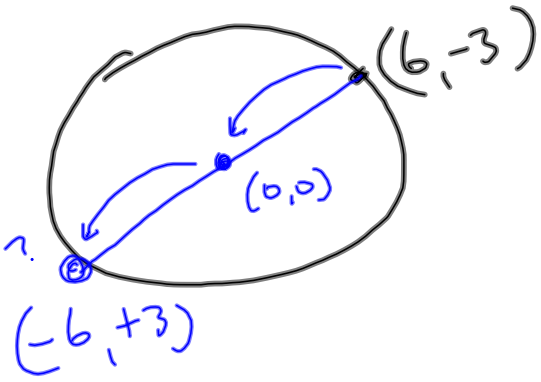
$x^2+y^2=25$

$x^2+y^2=?$

$(3)^2+(-4)^2 = 9+16=25$

$x^2+y^2=25$

p.229
Q26

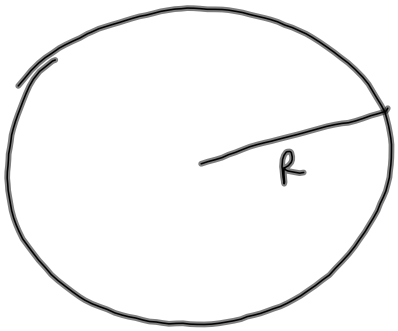


$x^2+y^2=45$

Centre $(0,0)$ ✓

Radius $=\sqrt{45}$

229
Q27



$x^2+y^2=40$

Centre $(0,0)$

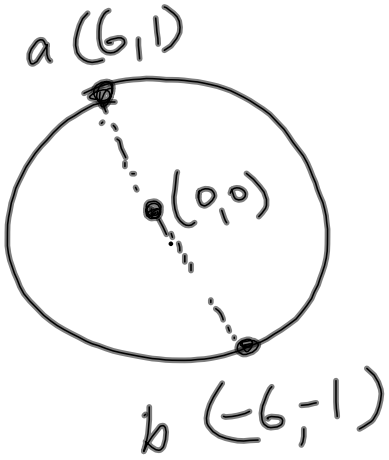
Radius $=\sqrt{40}$

$A=\pi R^2$

$A=(\sqrt{40})^2\pi$

$=40\pi$

Q25



$$x^2+y^2 = 6^2+1^2$$

$$x^2+y^2 = 37$$

p.231 Q11

circle?

C(1,2) contains (2,5)

r = ?

$$r = \sqrt{(1)^2+(3)^2} = \sqrt{10}$$

$$(x-1)^2 + (y-2)^2 = 10$$

p.131 Q12

Circle?

$C(2,-1)$ with $(6,4)$

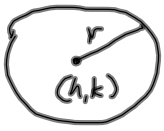
$R = \sqrt{(4)^2+(5)^2} = \sqrt{16+25} = \sqrt{41}$

Circle: $(x-2)^2+(y+1)^2 = 41$

$(6-2)^2+(4+1)^2 =$

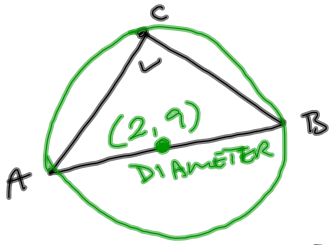
$4^2+5^2 = 41$

$(x-h)^2+(y-k)^2 = r^2$



$A(-1,5)$ $B(5,13)$ $C(-2,12)$

Show Right angle at C.



$m_{AC} = \frac{12-5}{-2+1} = \frac{7}{-1} = -7$

$m_{BC} = \frac{12-13}{-2-5} = \frac{-1}{-7} = \frac{1}{7}$

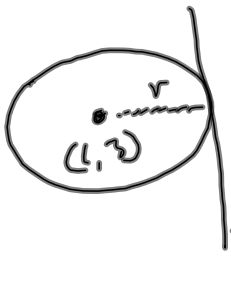
$\frac{1}{7}x - 7 = -1 \Rightarrow \perp$

$d = r = \sqrt{3^2+4^2} = 5$

$(x-2)^2+(y-9)^2 = 25$

30

$(1,3)$ tangent $3x+4y+10=0$

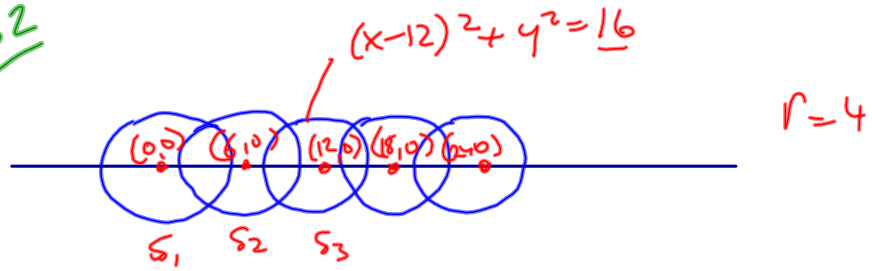


$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$r = \frac{|3(1)+4(3)+10|}{\sqrt{3^2+4^2}} = \frac{25}{5} = 5$$

$$(x-1)^2+(y-3)^2=25$$

32



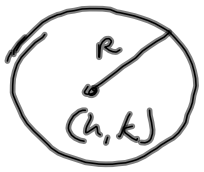
S_1: $x^2+y^2=16$

S_2: $(x-6)^2+y^2=16$

S_3 $(x-12)^2+y^2=16$

S_4 $(x-18)^2+y^2=16$

$$(x-h)^2 + (y-k)^2 = R^2$$



$$1x^2 + 1y^2 + 2gx + 2fy + c = 0$$

centre $(-g, -f)$

Radius $= \sqrt{|g|^2 + |f|^2 - c}$

Q17

$$2x^2 + 2y^2 - 2x - 6y - 13 = 0$$

$$x^2 + y^2 - x - 3y - \frac{13}{2} = 0$$

Centre $= \left(\frac{1}{2}, \frac{3}{2}\right)$

Radius $= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 - \left(-\frac{13}{2}\right)}$

$$= \sqrt{\frac{1}{4} + \frac{9}{4} + \frac{26}{4}}$$

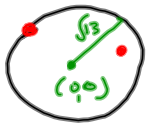
$$= \sqrt{\frac{36}{4}} = \sqrt{9} = 3$$

On, Inside, Outside?

P.235
Q24
Q25

eg.. $x^2 + y^2 = 13$

pt $(+3, -2)$



$(3,2)$ "On" eg..

$(3)^2 + (-2)^2 = ?$

$9 + 4 = 13$ yes. On

$(0,5)$ "Outside" eg

$(0)^2 + (5)^2 = 25 > 13$ Outside

$(0,2)$ Inside eg

$(0)^2 + (2)^2 = 4 < 13$ inside

Q23 $(4,-1)$

$x^2 + y^2 + 6x - 4y - 3 = 0$ RHS

$(4)^2 + (-1)^2 + 6(4) - 4(-1) - 3 = ?$

$16 + 1 + 24 + 4 - 3 = 42 > 0$

Outside

$(-3,2)$ $R = \sqrt{3^2 + 2^2 + 3}$

$= \sqrt{16} = 4$

Q. 27
p. 236

$C: x^2 + y^2 + 2x + 2y - 32 = 0$

$(-4, k) \in C$. Find 2 k values?

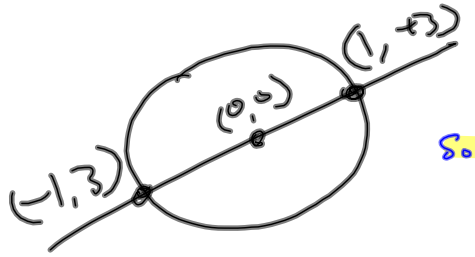
$$(-4)^2 + (k)^2 + 2(-4) + 2(k) - 32 = 0$$

$$16 + k^2 - 8 + 2k - 32 = 0$$

$$k^2 + 2k - 24 = 0$$

$$(k - 4)(k + 6) = 0$$

$$k = 4 \text{ or } -6$$



p. 239
Q. 2

Solve?

1 Rewrite

$x + 2y - 5 = 0 \Rightarrow x = 5 - 2y$

$x^2 + y^2 = 10$

Sub in and solve quadratic

2 $(5 - 2y)^2 + y^2 = 10$

$25 + 4y^2 - 20y + y^2 = 10$

$5y^2 - 20y + 15 = 0$

$y^2 - 4y + 3 = 0$

$(y - 3)(y - 1) = 0$

$y = 3$ and 1

Put back into Line

3 $x = 5 - 2(3) = -1$ $(-1, 3)$

$x = 5 - 2(1) = 3$ $(3, 1)$

Q5 $x-y-1=0 \Rightarrow x=y+1$ P.239

$x^2+y^2-2x-2y+1=0$

.... $\Rightarrow 2y^2-2y=0$

$y^2-y=0$

HCF $y(y-1)=0$

$y=0$ | $y-1=0$, $y=1$

$x=0+1=1$ $(1,0)$

$x=1+1=2$ $(2,1)$ ✓

P.239 Q6 $x-2y-1=0 \Rightarrow x=2y+1$

$x^2+y^2+2x-8y-8=0$

$\Rightarrow 5y^2-5=0$

DIFF 2 Squares $y^2-1=0 \longrightarrow y^2=1$

$y=\pm 1$

$(y+1)(y-1)=0$

$y+1=0$, $y=-1$ | $y-1=0$, $y=1$

$x=2(-1)+1=-1 \Rightarrow (-1,-1)$

$x=2(1)+1=3 \Rightarrow (3,1)$

P.238 eg

$L: 3x - y + 8 = 0$

$C: x^2 + y^2 - 4x - 8y + 10 = 0$

Prove L is a tangent?

$c = (2, 4)$ $\quad r = \sqrt{2^2 + 4^2 - 10} = \sqrt{10}$

$$|ct| = \frac{|3(2) - 4 + 8|}{\sqrt{(3)^2 + (-1)^2}} = \frac{10}{\sqrt{10}} = \sqrt{10}$$

$\Rightarrow$ yes it is tangent since $|ct| = r$

$L: 5x - 3y - 17 = 0$

$C: x^2 + y^2 = 17$

$c = (0, 0)$ $\quad r = \sqrt{17}$

$$d = |ct| = \frac{|5(0) - 3(0) - 17|}{\sqrt{5^2 + 3^2}} = \frac{17}{\sqrt{34}} \neq r$$

not tangent!

a (2,2) b (6,4) c (4,8)

243
Q1-3

$x^2 + y^2 + 2gx + 2fy + c = 0$

(a) → $4 + 4 + 2g2 + 2f2 + c = 0$

$8 + 4g + 4f + c = 0$

$4g + 4f + c = -8$

Q3
246

$x^2 + y^2 + 2gx + 2fy + c = 0$ contains

(4,3) & (6,-3) , $3x - y - 7 = 0$ Contains centre.

$4^2 + 3^2 + 2g(4) + 2f(3) + c = 0$

$25 + 8g + 6f + c = 0$

① $8g + 6f + c = -25$

$6^2 + (-3)^2 + 2g(6) + 2f(-3) + c = 0$

$45 + 12g - 6f + c = 0$

② $12g - 6f + c = -45$

$3(-g) - (-f) - 7 = 0$

③ $-3g + f = 7$

$8g + 6f + c = -25$
$-12g + 6f - c = 45$
$-4g + 12f = 20$

④ $-g + 3f = 5$

$-9g + 3f = 21$
$\pm g \mp 3f = \pm 5$
$-8g = 16$

$g = -2$

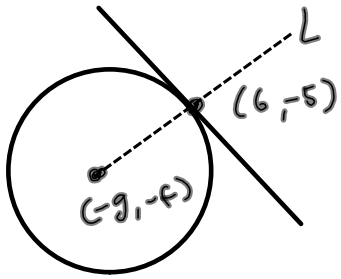
$-3(-2) + f = 7$
$6 + f = 7$
$f = 1$

$8(-2) + 6(1) + c = -25$
$-10 + c = -25$
$c = -15$

Q6 p.246

$(4,1)$ & $(6,-5)$ on circle

T: $2x-y-17=0$ Tangent at $(6,-5)$



$m_T = \frac{-2}{-1} = 2$ $\perp$ $-\frac{1}{2}$

$y-y_1 = m(x-x_1)$

$y+5 = -\frac{1}{2}(x-6)$

$2y+10 = -x+6$

$L: x+2y+4=0$

etc...

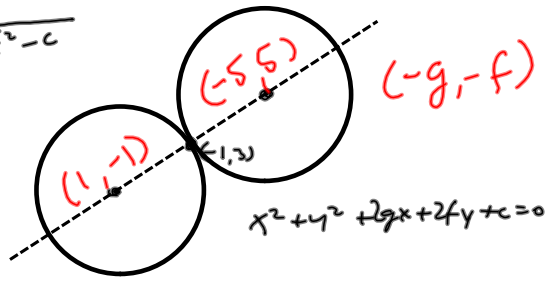
SOLN: $g=-2$, $f=3$, $c=-7$

p.247

$r=\sqrt{20}$ Contains $(-1,3)$

Centre on: $x+y=0$

$\sqrt{g^2+f^2-c}$



$x^2+y^2+2gx+2fy+c=0$

$\sqrt{20} = \sqrt{g^2+f^2-c}$

$20 = g^2+f^2-c$

$-g-f=0$

$g+f=0$ $\quad g=-f$

$(-1)^2+3^2+2g(-1)+2f(3)+c=0$

$10-2g+6f+c=0$

$-2g+6f+c=-10$

$20=(-f)^2+f^2-c = 2f^2-c=20$

$-2(-f)+6f+c = -10 = 8f+c$

$2f^2-c=20$

$8f+c=-10$

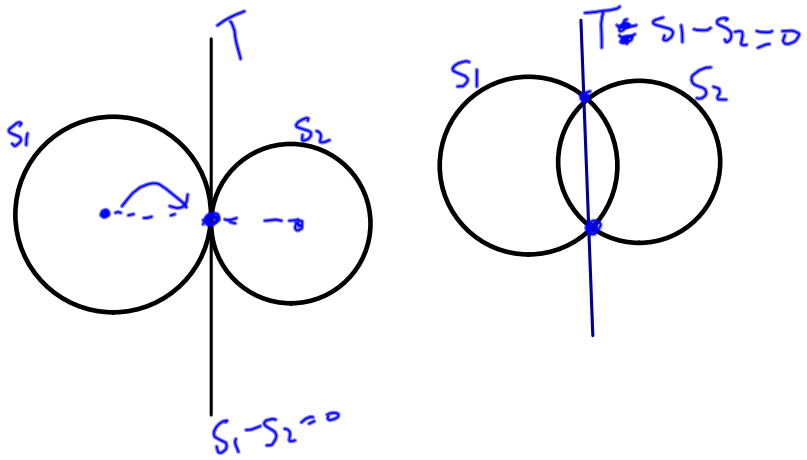
$2f^2+8f=10$

$f^2+4f-5=0$

$(f+5)(f-1)=0$

$f=-5$ or 1

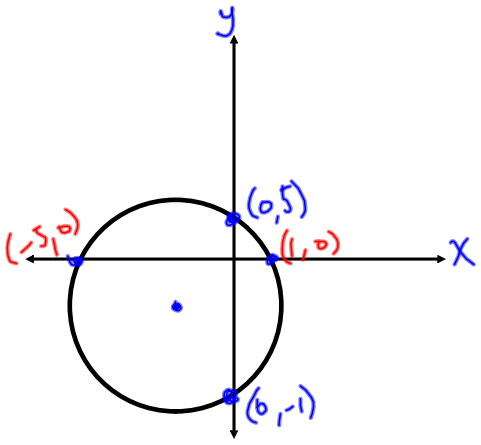
$\Rightarrow g=5$ or -1



Q) $(x+2)^2 + (y-2)^2 = 13$

$C = (-2, 2)$ $R = \sqrt{13}$

pts of axis intersection



at y axis $x = 0$

$\Rightarrow (0+2)^2 + (y-2)^2 = 13$

$4 + y^2 + 4 - 4y = 13$

$y^2 - 4y - 5 = 0$

$(y-5)(y+1) = 0$

$y = 5$ or -1

pts: $(0,5)$ and $(0,-1)$

at x axis $y = 0$

$\Rightarrow (x+2)^2 + (0-2)^2 = 13$

$\Rightarrow x^2 + 4x + 4 + 4 = 13$

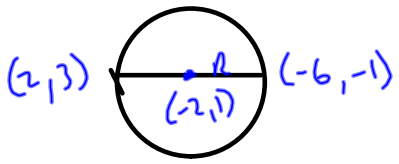
$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5$ or 1

pts $(-5,0)$ & $(1,0)$

Q3 $(2,3), (-6,-1)$



circle?

midpt = $(-2, 1)$ = centre

Radius = $\sqrt{(4)^2+2^2} = \sqrt{20}$

Circle: $(x+2)^2+(y-1)^2=20$

Cuts y axis pa Q.
(PQ⊥?

at y axis $x=0$

$\Rightarrow (0+2)^2+(y-1)^2=20$

$4+y^2+1-2y=20$

$y^2-2y-15=0$

$(y-5)(y+3)=0$

$y=5$ or -3

pts $(0,5), (0,-3)$

distance $P(0,5)$ and $Q(0,-3)$

$d=8$