The product of the first three terms of a geometric sequence is 27 and their sum is 13. Find the first four terms of the sequence.

let $T_1 = a$

Ratio $= r$

a, ar, ar^2

PRODUCT

$(a)(ar)(ar^2) = 27$

$a^3r^3 = 27$

$(ar)^3 = 27$

$ar = 3 \Rightarrow a = 3/r$ ①

SUM

$a + ar + ar^2 = 13$ ②

① → ②

$\frac{3}{r} + \frac{3}{r}r + \frac{3}{r}r^2 = 13$

$3/r + 3 + 3r = 13$

$3/r + 3r - 10 = 0$

$3r^2 - 10r + 3 = 0$

$(3r - 1)(r - 3) = 0$

$r = 1/3, \quad r = 3$

If $r = 1/3 \Rightarrow a = 9 \Rightarrow 9, 3, 1, 1/3$

if $r = 3 \Rightarrow a = 1 \Rightarrow 1, 3, 9, 27$

S1. P.1 Q4

Sequences and Series question

Prove by induction

$S_n = \frac{a(1-r^n)}{1-r}$ $\quad T_n = ar^{n-1}$

n=1

$T_1 = S_1 = \frac{a(1-r)}{1-r} = a$ ✓

Steps

① Show n=1

② Assume n=K

③ Prove n=K+1

④ Conclude

Assume true n=K

$S_K = \frac{a(1-r^K)}{1-r}$

To prove for n=K+1

RHS

$S_{K+1} = \frac{a(1-r^{K+1})}{1-r} = S_K + T_{K+1}$

RHS $= \frac{a(1-r^K)}{1-r} + ar^{K+\cancel{1}-\cancel{1}}$

$= a\left(\frac{1-r^K}{1-r} + \frac{r^K}{1}\right)$

$= a\left(\frac{1-\cancel{r^K}+\cancel{r^K}-r^{K+1}}{1-r}\right)$

QED

$\Rightarrow$ It's true for n=1, n=k, n=k+1, and all $n \in \mathbb{N}$

$\lim_{n\to\infty} \frac{a}{n} = 0$

divide above and below by n!

Limits

S1.P1.Q4

$$\lim_{n\to\infty} \frac{2n}{3n+4} =$$

$$= \lim_{n\to\infty} \frac{2}{3+\underbrace{\frac{4}{n}}_{=0}} = \frac{2}{3}$$

$|a| < 1$

$\left(\frac{1}{a}\right)^{\infty} = 0$

Limits

S.1.P.1.Q4

$$\lim_{n\to\infty} 2\left(\frac{1}{3}\right)^n = 0$$

$a > 1$

$a^{\infty} = \infty$

Limits

S1 P1 Q4

$$\lim_{n \to \infty} \frac{1}{3}(2)^n = \infty$$

De Moivre's Theorem Question

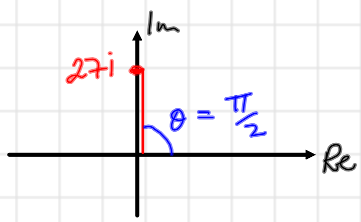
Find the cube roots of $27i$.

$$z^3 = 27i \qquad z = (27i)^{\frac{1}{3}}$$

Polar form

$r = ?$

$\theta = ?$



$r = 27$

Polar $=$ $\quad z = 27\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

general polar $=$ $\quad z = 27\left(\cos\left(\frac{\pi}{2} + 2n\pi\right) + i\sin\left(\frac{\pi}{2} + 2n\pi\right)\right)$

deMoivre $\quad z = 27^{\frac{1}{3}}\left(\cos\left(\frac{\frac{\pi}{2} + 2n\pi}{3}\right) + i\sin\left(\frac{\frac{\pi}{2} + 2n\pi}{3}\right)\right)$

Cube root both sides $\rightarrow$ $\quad z^{\frac{1}{3}} = 3\left(\cos\frac{\pi}{3}\left(\frac{1}{2} + 2n\right) + i\sin\frac{\pi}{3}\left(\frac{1}{2} + 2n\right)\right)$

3 Solutions

$n = 0$ $\quad z_1 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$n = 1$ $\quad z_2 = 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$n = 2$ $\quad z_3 = 3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = 0 - 3i$