Polar Form

Multiply?

eg.. $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$w = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$z.w = ?$

$zw = 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Rule:
"multiply moduli
add angles"

S2P1Q2

Divide?

$\frac{z}{w} = ?$

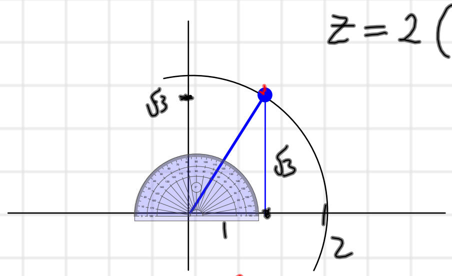
$\frac{z}{w} = \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

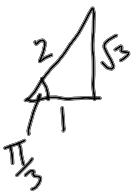
Rule:
"divide moduli
subtract angles"

$|z| = 2$

$\theta = \frac{\pi}{3}$

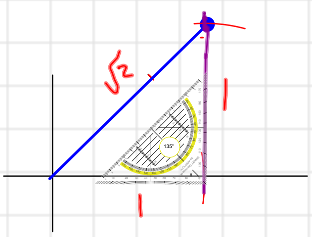
$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

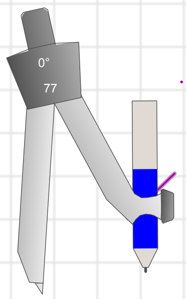


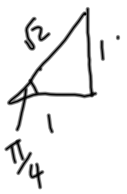


Construct √3 and √2 on LCHL course

Draw line of length √2 ?







Related:
$\left.\begin{array}{l}\cos 2\theta \\ \sin 2\theta \\ \sin 3\theta\end{array}\right\} = ?$

de Moivre

$r(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$

$(a+b)^3 = a^3 + 3a^2b + 3b^2a + b^3$

$i^2 = -1$, $i^3 = -i$

Re = Re

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

Show $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ using de moivres theorem.

LHS RHS

$(\cos\theta + i\sin\theta)^3 = \underbrace{\cos 3\theta}_{\text{Real}} + i\underbrace{\sin 3\theta}_{\text{imaginary}}$

expand

$LHS = \cos^3\theta + 3i\cos^2\theta\sin\theta + 3i^{\cancel{2}}\cos\theta\sin^2\theta + i^3\sin^3\theta$

$= \underbrace{(\cos^3\theta - 3\cos\theta\sin^2\theta)}_{\text{Real}} + \underbrace{(3i\cos^2\theta\sin\theta - i\sin^3\theta)}_{\text{imaginary}}$

$\cos 3\theta = \cos^3\theta - 3\cos\theta\,\underline{\sin^2\theta}$

$= \cos^3\theta - 3\cos\theta(1 - \cos^2\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

23-1-2013

HW

Using de Moivres theorem

1. Show $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
2. show $\cos 2\theta = 2\cos^2\theta - 1$
3. show $\sin 2\theta = 2\cos\theta\sin\theta$

23-1-2013
HW

de Moivre
$r(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$

$(a+b)^3 = a^3+3a^2b+3b^2a+b^3$
$i^2=-1,\ i^3=-i$

$Im = Im \Rightarrow$

$1 = \cos^2\theta + \sin^2\theta$
$\cos^2\theta = 1-\sin^2\theta$

① Show $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ using de Moivres theorem.

$$\underbrace{(\cos\theta + i\sin\theta)^3}_{LHS} = \underbrace{\underbrace{\cos 3\theta}_{Real} + \underbrace{i\sin 3\theta}_{imaginary}}_{RHS}$$

expand
$$LHS = \cos^3\theta + 3i\cos^2\theta\sin\theta + 3i^2\cos\theta\sin^2\theta + i^3\sin^3\theta$$
$$= \underbrace{(\cos^3\theta - 3\cos\theta\sin^2\theta)}_{Real} + \underbrace{(3i\cos^2\theta\sin\theta - i\sin^3\theta)}_{imaginary}$$

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$
$$= 3(1-\sin^2\theta)\sin\theta - \sin^3\theta$$
$$= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$$
$$= 3\sin\theta - 4\sin^3\theta$$

23-1-2013
HW

using de Moivre's theorem
② show $\cos 2\theta = 2\cos^2\theta - 1$
and ③ show $\sin 2\theta = 2\cos\theta\sin\theta$

$$\underbrace{(\cos\theta + i\sin\theta)^2}_{LHS} = \underbrace{\underbrace{\cos 2\theta}_{Real} + \underbrace{i\sin 2\theta}_{imaginary}}_{RHS}$$

expand LHS
$(a+b)^2 = a^2+2ab+b^2$

$$\cos^2\theta + 2i\cos\theta\sin\theta + i^2\sin^2\theta = RHS$$

$$\underbrace{(\cos^2\theta - \sin^2\theta)}_{Real} + \underbrace{(2i\cos\theta\sin\theta)}_{imaginary} = RHS$$

$Re = Re$

$1 = \cos^2\theta + \sin^2\theta$
$\sin^2\theta = 1-\cos^2\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - (1-\cos^2\theta)$$
$$= \cos^2\theta - 1 + \cos^2\theta$$
$$= 2\cos^2\theta - 1$$

$Im = Im$

$$\sin 2\theta = 2\cos\theta\sin\theta$$