Complex Numbers

Revision

$z_1 = 3+2i \qquad z_2 = 4-6i$

① $z_1 + z_2 = 7 - 4i$

② $z_1 - z_2 = -1 + 8i$

③ $3z_1 = 9 + 6i \qquad 3z_2 = 12 - 18i$

④ $\bar{z}_1 = 3 - 2i$

⑤ $z_1 . z_2 = (3+2i)(4-6i) = 3(4-6i) + 2i(4-6i)$

$= 12 - 18i + 8i \overset{+}{-} 12\cancel{i^2}$

$= 24 - 10i$

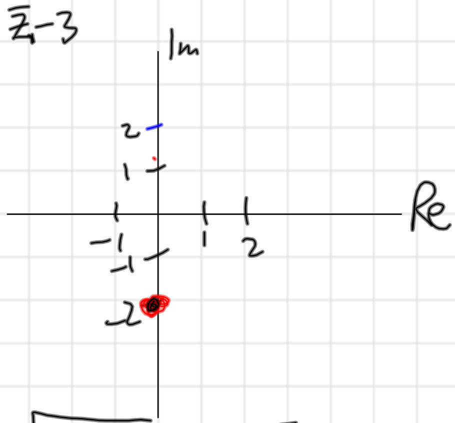
⑥ $\frac{z_2}{z_1} = \frac{(4-6i)(3-2i)}{(3+2i)(3-2i)} = \frac{12 - 8i - 18i \mp 12\cancel{i^2}}{9 \pm 4\cancel{i^2}}$

$= -26i/13 = -2i$

⑦ Diagram $\bar{z}_1 - 3$

$\bar{z}_1 = 3 - 2i$

$\bar{z}_1 - 3 = -2i$



modulus

$|a+bi| = \sqrt{a^2+b^2}$

⑧ $|z_1| = \sqrt{3^2+2^2} = \sqrt{13}$

⑨ $|z_2| = \sqrt{4^2+6^2} = \sqrt{52}$

Section 3.2 Complex numbers

Example 1

Solve the equation $x^2 + 25 = 0$.

Example 2

Solve the equation $x^2 + 2x + 2 = 0$.

Example 3

If $z_1 = 2 + 3i$, $z_2 = 3 - 4i$ and $z_3 = 1 + 5i$, express each of the following complex numbers in the form $a + bi$.

(i) $z_1 + z_3$ (ii) $z_2 . z_3$ (iii) $z_1(z_2 + z_3)$

Section 3.3 **Division of complex numbers**

Example 1

Write $\frac{3+4i}{2-5i}$ in the form $a+bi$.

Example 2

Find x and y if $x + 2i + 2(3 - 5yi) = 8 - 13i$.

$x + 2i + 6 - 10yi = 8 - 13i$

$(6+x) + (2-10y)i = 8 - 13i$

Re=Re $\quad 6 + x = 8 \Rightarrow x = 2$

Im=Im $\quad 2 - 10y = -13$

$\frac{-10y}{-10} = \frac{-15}{-10}$

$y = 3/2$

Example 3

Given that $(z + 1)(2 - i) = 3 - 4i$, find z in the form $x + yi$, where $x, y \in R$.

Example 4

Express $\sqrt{5 + 12i}$ in the form of $a + bi$, where $a, b \in R$.

Section 3.4 Argand diagram – Modulus

Example 1

Given that $z_1 = 4 + i$ and $z_2 = -2 + 2i$, plot the following on an Argand diagram: