# Revision

The Line and

# The Circle

14 Jan 2013

| | | | |
|---|---|---|---|
| **2.2 Co-ordinate geometry** | – use slopes to show that two lines are<br>• parallel<br>• perpendicular<br>– recognise the fact that the relationship $ax + by + c = 0$ is linear<br>– solve problems involving slopes of lines | – calculate the area of a triangle<br>– recognise that $(x\text{-}h)^2 + (y\text{-}k)^2 = r^2$ represents the relationship between the $x$ and $y$ co-ordinates of points on a circle centre $(h, k)$ and radius $r$<br>– solve problems involving a line and a circle with centre (0, 0) | – solve problems involving<br>• the perpendicular distance from a point to a line<br>• the angle between two lines<br>– divide a line segment internally in a given ratio m:n<br>– recognise that $x^2+y^2 +2gx+2fy+c = 0$ represents the relationship between the $x$ and y co-ordinates of points on a circle centre $(-g,-f)$ and radius r where $r = \sqrt{(g^2+f^2 - c)}$<br>– solve problems involving a line and a circle |

Coordinate geometry of the circle assumes knowledge of coordinate geometry of the line.

# Coordinate Geometry of The Line

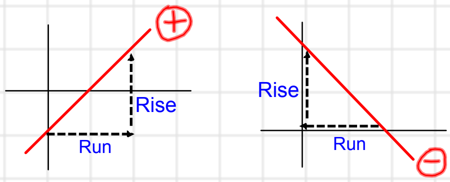
from Junior Cent

distance between 2 points: $|ab| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint (average point): midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

slope (given 2 points): $m = \frac{y_2-y_1}{x_2-x_1}$

slope (given graph): $m = \frac{\text{Rise}}{\text{Run}}$




Slope from equation of line
slope (given: y = mx + c): $m=m$

Slope from equation of line
slope (given ax + by+ c = 0): $m = \frac{-a}{b}$

parallel slopes: $\Rightarrow\ m_1 = m_2$

perpendicular slopes: $\Rightarrow\ m_1 \times m_2 = -1$

$\frac{a}{b} \perp -\frac{b}{a}$ if $m_1 = \frac{a}{b} \Rightarrow m_2 = -\frac{b}{a}$

from Junior Cent

equation of a line: $y-y_1 = m(x-x_1)$

Graph line
(given ax + by+ c = 0): let $x=0$, find $y$
let $y=0$, find $x$ ....

Graph line
(given: y = mx + c): $m = \frac{\text{Rise}}{\text{Run}}$ $c =$ y-intercept

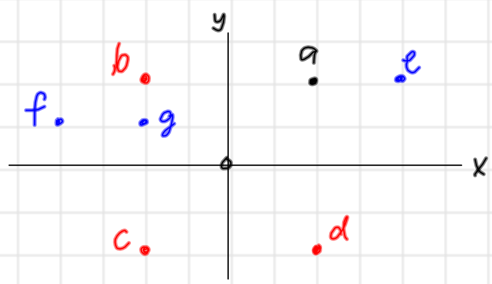
point on a line: If $(x_1, y_1)$ is on the line $ax + by + c = 0$
then $ax_1 + by_1 + c = 0$

point of intersection of 2 lines: Solve simultaneous equations

# Transformations

Image of $a$ under...

axial symmetry
$S_x \rightarrow d$
$S_y \rightarrow b$
central symmetry
$S_0 \rightarrow c$
transformation
$\vec{fg} \rightarrow e$



# Coordinate Geometry of The Line

Leaving Cert. Ordinary Level

**Area of a circle with vertex (0, 0)**

$$A = \frac{1}{2}\left|x_1y_2 - x_2y_1\right|$$

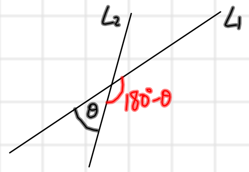
**Area of a circle (0, 0) not vertex**

⇒ transform triangle so it has a vertex at (0, 0)

Leaving Cert. Higher Level

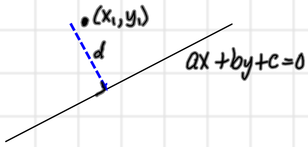
**The angle between 2 lines**

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

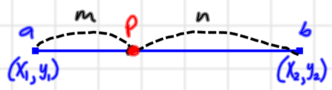



**The perpendicular distance from a point to a line**

$$d = \frac{\left|ax_1 + by_1 + c\right|}{\sqrt{a^2 + b^2}}$$




**Internally divide a line segment in the ratio m:n**




can use ratios or formula

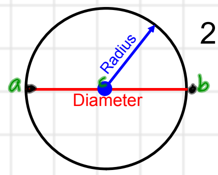
$$p = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

# The Circle

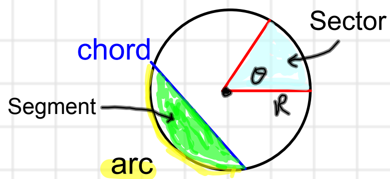
Junior Cert. Geometry

Diameter, radius

Centre is midpoint of diameter




2r = d




c is midpoint of [ab]

Area & Circumference

Area = $\pi R^2$

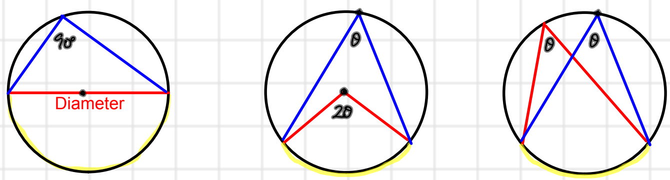
Circumference $= 2\pi R$

Area of sector $= \left(\frac{\theta}{360°}\right)\pi r^2$

Arc length $= \left(\frac{\theta}{360°}\right)2\pi R$

Angles standing on same arc

Angle of 90° stands on diameter




Perpendicular line from centre through chord bisects chord

Tangent is perpendicular to line [ct]

## The Circle

Leaving Cert. Ordinary Level

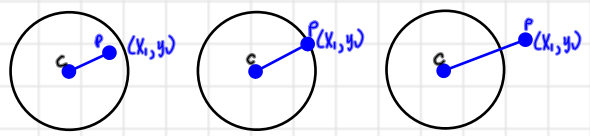
(1) **Equation of a circle**

Centre $(0,0)$, Radius $r$: $x^2 + y^2 = r^2$

Centre $(h,k)$, Radius $r$: $(x-h)^2 + (y-k)^2 = r^2$

(2) **Points in, on, outside a circle**

| Point inside circle | Point on circle | Point outside circle |
|---|---|---|

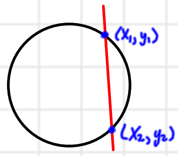



| $\lvert cp \rvert < r$ | $\lvert cp \rvert = r$ | $\lvert cp \rvert > r$ |
|---|---|---|
| $(x_1-h)^2+(y_1-k)^2 < r^2$ | $(x_1-h)^2+(y_1-k)^2 = r^2$ | $(x_1-h)^2+(y_1-k)^2 > r^2$ |

Sub. point into the equation

(3) **Intersection of a line a circle**

Solve simultaneous equations




- rewrite linear
- sub into circle
- solve quadratic
- sub solutions into linear

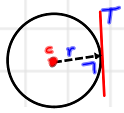
(4) **Circle intersecting axes**

Intersects x-axis where y=0

Intersects y-axis where x=0

(5) **Proving a line is a tangent to a circle**

⇒ only 1 point of intersection




also distance from c to T is r

## The Circle

Leaving Cert. Higher Level

(6) **General equation of a circle**

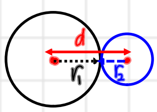
$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $= (-g, -f)$

Radius $= \sqrt{g^2 + f^2 - c}$

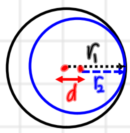
(7) **Touching circles**

Touch externally



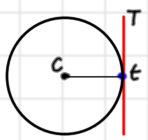

$d = r_1 + r_2$

Touch internally




$d = r_1 - r_2$

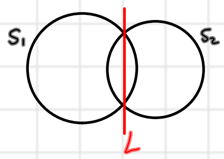
(8) **Equation of tangent**




- find slope [ct]
- find slope T
- $y - y_1 = m(x - x_1)$

or use formula in log tables

(9) **Common chord (or tangent)**




circles $s_1$ and $s_2$ expressed in the form

$x^2 + y^2 + 2gx + 2fy + c = 0$

then $S_1 - S_2 = L$

(10) **Circles touching axes**

touches x-axis

$\Rightarrow g^2 = c$ and $r = |-f|$

touches y-axis

$\Rightarrow f^2 = c$ and $r = |-g|$