r = modulus

$|a+bi| = \sqrt{a^2+b^2}$

θ = argument

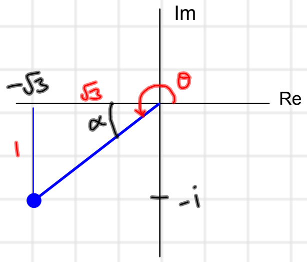
$x + iy$

$r(\cos\theta + i\sin\theta)$

Example 4

Write $-\sqrt{3} - i$ in polar form.

$r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$



$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$

$\theta = \pi + \pi/6 = 7\pi/6$

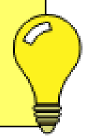
$-\sqrt{3} - i = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

de Moivres Theorem

Remember...

$[r(\cos\theta + i\sin\theta)]^n =$

$r^n(\cos n\theta + i\sin n\theta)$



de Moivre's Theorem

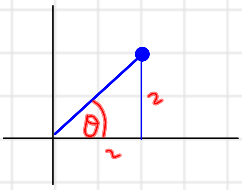
evaluate $(2+2i)^4$

① Write in Polar Form

$r=?$

$\theta=?$

$r = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$



$\theta = \pi/4$

Polar form

$(2+2i)^4 = \left[2\sqrt{2}\left(\cos \pi/4 + i\sin \frac{\pi}{4}\right)\right]^4$

② Apply deMoivres theorem

$[r(\cos\theta + i\sin\theta)]^n$

$= r^n(\cos n\theta + i\sin n\theta)$

$= (2\sqrt{2})^4\left(\cos\left(\frac{\cancel{4}\pi}{\cancel{4}}\right) + i\sin\left(\frac{\cancel{4}\pi}{\cancel{4}}\right)\right)$

③ Put back into Cartesian form

$= 64(-1+0i)$

$= -64$

Homework:

use de Moivre's theorem to evaluate

① $(1+i)^4$

② $(\sqrt{3}+i)^4$

③ $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^4$