Differentiation

Differentiation	
$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
e^x	e^x
$\ln x$	$\frac{1}{x}$

Using Chain Rule	
$f(x)$	$f'(x)$
u^n	$nu^{n-1}.\frac{du}{dx}$
$\sin u$	$\cos u.\frac{du}{dx}$
$\cos u$	$-\sin u.\frac{du}{dx}$
$\tan u$	$\sec^2 u.\frac{du}{dx}$
e^u	$e^u.\frac{du}{dx}$
$\ln u$	$\frac{1}{u}.\frac{du}{dx}$

Inverse Trigonometric	
$f(x)$	$f'(x)$
$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2-x^2}}$
$\tan^{-1}\frac{x}{a}$	$\frac{a}{a^2+x^2}$
$f(x)$	$f'(x)$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}}.\frac{du}{dx}$
$\tan^{-1} u$	$\frac{1}{1+u^2}.\frac{du}{dx}$

Differentiation of a constant $= 0$

Differentiation by Rule
Multiply by the power and reduce the power by 1.

$y = x^n$

$\frac{dy}{dx} = nx^{n-1}$

Remember

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{1}{x^n} = x^{-n}$

$\frac{1}{\sqrt[n]{x^m}} = x^{-\frac{n}{m}}$

Chain Rule
Bring the power down to the front, reduce the power by 1 and multiply by the derivative of the bracket.

$y = u^n$

$\frac{dy}{dx} = nu^{n-1}.\frac{du}{dx}$

Implicit Differentiation

1. Differentiate term by term, on both sides with respect to x.
2. Bring all the terms with $\frac{dy}{dx}$ to the left and bring all the other terms to the right.
3. Make $\frac{dy}{dx}$ the subject of the equation by factorising and dividing across where necessary
4. When we differentiate y we get $\frac{dy}{dx}$

Given that $2x^3 + 3xy^2 - y^3 + 6 = 0$ evaluate $\frac{dy}{dx}$

$$6x^2 + \left(3x(2y)\left(\frac{dy}{dx}\right) + y^2(3)\right) - 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6x^2 - 3y^2}{6xy - 3y^2}$$

Parametric Equation

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}.\frac{dt}{dx}$$

$x = 4t^3$
$y = (1+3t^2)^2$

$\frac{dx}{dt} = 12t^2$

$\frac{dy}{dt} = 2(1+3t^2)(6t)$

$\frac{dy}{dt} = 12t(1+3t^2)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{12t(1+3t^2)}{12t^2}$$

$$\frac{dy}{dx} = \frac{1+3t^2}{t}$$

Product Rule
First by the derivative of the Second plus Second by the derivative of the first.

$y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{dv}{dx}$

Quotient Rule
Bottom by the derivative of the top minus Top by the derivative of the bottom all over the Bottom squared

$y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{du}{dx}}{v^2}$$

Graph Applications

Turning Points

Solve $\frac{dy}{dx} = 0$ then put x value into $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} > 0$	**minimum**
$\frac{d^2y}{dx^2} < 0$	**maximum**
$\frac{d^2y}{dx^2} = 0$	**point of inflection**

Nature of the curve

$\frac{dy}{dx} > 0$	**increasing**
$\frac{dy}{dx} < 0$	**decreasing**

Asymptote

Vertical Asymptote	Bottom $= 0$
Horizontal Asymptote	$y = \lim_{x\to\infty} f(x)$

Differentiation from 1st Principles

$f(x) =$

$f(x+h) =$

$f(x+h) - f(x) =$

$$\frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} =$$

Newton Rhapson

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If x_n is an approximate solution of the equation $f(x) = 0$ then x_n is a better approximation.

Show that $1 - 3x - x^3 = 0$ *has a real root between 0 and 1.*

$f(x) = 1 - 3x - x^3$
$f(0) = 1 - 3(0) - (0)^3 = 1$
$f(1) = 1 - 3(1) - (1)^3 = -3$
Since $f(x)$ changes sign between 0 and 1, the graph cross the x axis and therefore a root lies between 0 and 1.

Starting with $x_1 = 0$ *as a first approximation of* $1 - 3x - x^3 = 0$*, use Newton-Rhapson to find* x_2*, the second approximation.*

$f(x) = 1 - 3x - x^3$
$f(0) = 1 - 3(0) - (0)^3 = 1$

$f'(x) = -3 - 3x^2$
$f'(0) = -3 - 3(0)^2 = -3$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x)_1}$$

$$x_2 = 0 - \frac{1}{-3}$$

$$x_2 = \frac{1}{3}$$

We could use this again with our formula to find x_3

Rates of Change

$\frac{dy}{dx}$ is called the 'rate of change of y with respect to x'.
It shows how changes in y are related to changes in x.
$\frac{dy}{dx} = 5$, then y is increasing 5 times as fast as x increases

Distance, Velocity & Acceleration

A particle moving along a straight line such that, after t seconds, the distance moved, s metres, is given by
$s = t^3 - 9t^2 + 15t - 3$

$\frac{ds}{dt} = velocity$
$\frac{d^2s}{dt^2} = acceleration$

Related Rates of Change

The radius of a circle increases at 4 cm/s. What is the rate of increase of the area when the radius is 5 cm.
Radius increases at 4cm/s. Therefore $\frac{dr}{dt} = 4$ and asked to find $\frac{dA}{dt}$ when $r = 5$.

$$\frac{dA}{dt} = \frac{dr}{dt} \times \frac{dA}{dr}$$

We need to find $\frac{dA}{dr}$

Well $A = \pi r^2$
therefore $\frac{dA}{dr} = 2\pi$

$\frac{dA}{dt} = 4 \times 2\pi r$
$\frac{dA}{dt} = 8\pi r$
at $t = 5$

$\frac{dA}{dt} = 40\pi$