NB $\ln = \log_e$ $\log = \log_{10}$

Question 4 **(25 marks)**

In a science experiment, a quantity $Q(t)$ was observed at various points in time t. Time is measured in seconds from the instant of the first observation. The table below gives the results.

t	0	1	2	3	4
$Q(t)$	2·920	2·642	2·391	2·163	1·957

Q follows a rule of the form $Q(t) = Ae^{-bt}$, where A and b are constants.

(a) Use any two of the observations from the table to find the value of A and the value of b, correct to three decimal places.

$Q(t) = Ae^{-bt}$ $\quad Q(0) = Ae^{-b(0)} \Rightarrow A = 2{\cdot}92$

$Q(1) = 2{\cdot}92\, e^{-b(1)} = 2{\cdot}642$

$\Rightarrow e^{-b} = \dfrac{2{\cdot}642}{2{\cdot}92}$

$-b = \log_e\left(\dfrac{2{\cdot}642}{2{\cdot}92}\right) = -0{\cdot}100$

$\Rightarrow b = 0{\cdot}1$

(b) Use a different observation from the table to verify your values for A and b.

$Q(t) = 2{\cdot}92\, e^{-0{\cdot}1t}$

$Q(2) = 2{\cdot}92\, e^{-0{\cdot}1(2)} = 2{\cdot}391$ ✓

$$-bt = -b(t-1) = -bt + b$$

(c) Show that $Q(t)$ is a constant multiple of $Q(t-1)$, for $t \geq 1$.

$$Q(t) = Ae^{-bt}$$
$$Q(t-1) = Ae^{-b(t-1)}$$
$$Q(t) = Ae^{-b(t-1)} \cdot e^{-b}$$
$$= Q(t-1) \cdot e^{-b}$$

QED

(d) Find the value of the constant k for which $Q(t+k) = \frac{1}{2}Q(t)$, for all $t \geq 0$.

Give your answer correct to two decimal places.

$$\cancel{A}e^{-b(t+k)} = \frac{1}{2}\cancel{A}e^{-bt}$$
$$\cancel{e^{-bt}} \cdot e^{-bk} = \frac{1}{2}\cancel{e^{-bt}}$$
$$e^{-bk} = \frac{1}{2}$$
$$-bk = \log_e \frac{1}{2} = -0.6931$$
$$k(0.1) = 0.6931$$
$$k = 6.93 \quad (2 dp)$$

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