Financial Maths

Section 5.2

Depreciation

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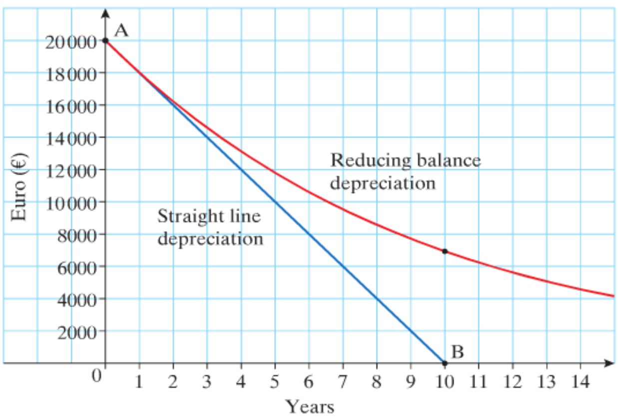
In the previous section, money lodged into a savings account appreciated in value. The future value was greater than the present value.

Depreciation occurs when the future value of an asset is less than the present value. Cars, computers and household appliances generally depreciate in value over time. Houses in Ireland appreciated in value up to 2007 but have since greatly depreciated in value relative to this "peak" value. Two types of depreciation can be considered.

1. **Straight line depreciation** occurs when the value of an object reduces by a constant amount each year.
   For example, take a car costing €20 000 that loses 10% of its original value each year. This car loses €2000 in value each year and so the car has no value after 10 years.
2. **Reducing balance depreciation** occurs when the value of an object reduces by a fixed percentage of its value each year.
   Consider a car costing €20 000 that loses 10% of its value each year on a reducing balance.
   The value of the car after 10 years $= €20\,000\,(1 - 0.1)^{10} = €6973.57$.



Depreciation: $F = P(1-i)^t$

F = future value
i = the percentage depreciation of €P per year
t = number of years
P = initial value

Example 1

A company buys a new machine priced at €35 000.
The machine depreciates by 20% on a reducing balance basis each year.

(i) What will the value of the machine be in 4 years time?
(ii) By how much has the machine depreciated in value during this time?

$F = P(1-i)^t$ (i) $F = 35\,000(0.8)^4 = €\,14,336$

$D = P - F$ (ii) $D = 35\,000 - 14\,336 = €\,20,664$

Example 2

A garage has a petrol stock of 100 000 litres.
If the manager estimates (a) that he will sell 4000 litres a day
(b) that he will sell 5% of his stock per day,
calculate the difference in his estimates after 20 days.

estimate (a) $100,000 - 4000(20) = 20,000$ litres

estimate (b)
$F = P(1-i)^t$

$F = 100,000(0.95)^{20}$

$= 35,848.59$ litres

Difference $= 35848.59 - 20\,000 = 15\,849$ litres

Exercise 5.2

(In the following exercise, depreciation is used on a reducing balance basis unless otherwise stated.)

1. How much will a car, costing €30 000, be worth in
(i) five years (ii) ten years time based on a depreciation of 15% per annum?

$F = P(1-i)^t$

$P = €30000$
$i = 15\%$ p.a. $\Rightarrow 1-i = 0{\cdot}85$

(i) $t = 5$ years

$F = 30\,000(0{\cdot}85)^5 = €13{,}311{\cdot}16$

(ii) $t = 10$ years

$F = 30\,000(0{\cdot}85)^{10} = €5906{\cdot}23$

2. A new television costs €1400. Assuming a depreciation rate of 8% per month, find the value of the television after 15 months.

$F = P(1-i)^t$

$P = €1400$
$i = 8\%$ p.m. $\Rightarrow 1-i = 0{\cdot}92$
$t = 15$ months
$F = ?$

$F = 1400(0{\cdot}92)^{15} = €400{\cdot}82$

3. A car costing €44 000 depreciates in value by 20% in the first year, and by 15% per year on a reducing balance basis for each subsequent year.
Find the value of the car after (i) 3 years (ii) 6 years.

$F = P(1-i)^t$

must calculate year 1 first

$P = €44\,000$

$i = 20\% \Rightarrow 1-i = 0.8$

$t = 1$ year

At end of year 1

$F = 44\,000(0.8)^1 = €35,200$

Years 2 and 3

$P = 35\,200$

$i = 15\% \Rightarrow 1-i = 0.85$

$t = 2$ years

Value after 3 years

$F = 35200\,(0.85)^2 = €25,432$

Years 2 to end of 6

$P = 35,200$

$t = 5$ years

$i = 15\% \Rightarrow 1-i = 0.85$

Value after 5 years

$F = 35200(0.85)^5 = €15,618.43$

4. A company buys a machine costing €140 000.
In order to facilitate its replacement, the company invests €25 000 in a bank offering a return of 3.5% per annum compound interest.
If the machine depreciates at a rate of 20% per annum, find
(a) (i) the value of the machine in 4 years time
(ii) the value of their savings investment in 4 years time.
(b) If inflation over the 4 years averages 2% per annum, find
(i) the cost of buying a new machine in 4 years time
(ii) how much money the company will need to add to their savings in order to replace the machine, taking the second-hand value of the machine in 4 years time into account.

(**Note:** Inflation is a rise in the *general level of prices* of goods and services in an economy.)

Depreciation

$F = P(1-i)^t$ a) i) $F = 140,000\,(0.8)^4 = €57,344$

Compound interest

$F = P(1+i)^t$ ii) $F = 25,000\,(1.035)^4 = €28,688.08$

inflation

$F = P(1+i)^t$ b) i) $F = 140,000(1.02)^4 = €151,540.50$

Assuming the 2nd Hand Value in 4 years time is not subject to inflation

ii) Difference between depreciated value and cost of new machine in 4 years.

$= 151,540.50 - 57344 = €94,196.50$

Extra money needed = Value difference Less savings

$= 94196.50 - 28688.05 = €65,508.45$

Extra investment = Present value of 65508.45 with discount rate of 4%

$P = \dfrac{A}{(1+i)^t}$

$P = 65508.45/(1.04)^4 = €55996.90$

5. A company asset reduces in value from €175 000 to €73 187.09, at a depreciation rate of 16% per annum over t years.

~~(i) By trial and error, estimate the value of t.~~

(ii) Using logs, find the value of t.

$F = P(1-i)^t$

$a = b^n$

$\Rightarrow n = \log_b a$

$P = 175\,000$

$F = 73\,187.09$

$i = 16\% \Rightarrow 1-i = 0.84$

$t = ?$

$$\Rightarrow \frac{F}{P} = (1-i)^t$$

Formula for t

$$\Rightarrow t = \log_{(1-i)} \frac{F}{P}$$

$$t = \log_{0.84}\left(\frac{73187.09}{175\,000}\right)$$

$$= 5 \text{ years}$$

6. A creamery has a stock of 60 000 kg of dried milk powder at the end of January 2004 If the stock is reduced at a rate of 15% per month, find the dried milk stock, to the nearest kg, at the beginning of April 2005.

$F = P(1-i)^t$

$P = 60\,000$ kg

$i = 15\%$ p.m. $\Rightarrow 1-i = 0.85$

$t = 15$ months

$$F = 60\,000(0.85)^{15} = 5241 \text{ kg}$$

7. A farmer buys a tractor for €180 000.
He assumes that the tractor will have a trade-in value of €80 000 in 10 years time.
(i) Calculate the rate of depreciation per annum, correct to one place of decimals, based on these figures.
(ii) At this rate, when will the value of the tractor fall below €60 000?

$F = P(1-i)^t$

$P = €\,180\,000$
$F = €\,80\,000$
$t = 10$ years
$i = ?$

$$80\,000 = 180\,000\,(1-i)^{10}$$

$$\frac{80\,000}{180\,000} = (1-i)^{10}$$

$$\sqrt[10]{\frac{4}{9}} = 1-i$$

$i = 1 - \sqrt[t]{\frac{F}{P}}$

$$i = 1 - \sqrt[10]{\frac{4}{9}} = 0.07789 \approx 7.8\%$$

7. A farmer buys a tractor for €180 000.
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(i) Calculate the rate of depreciation per annum, correct to one place of decimals, based on these figures.
(ii) At this rate, when will the value of the tractor fall below €60 000?

(ii)

$F = P(1-i)^t$

$P = €\,180\,000$
$F = €\,60\,000$
$i = 7.8\% \Rightarrow 1 - i = 0.922$

$\left.\right\}$ $t = ?$

$$60\,000 = 180\,000\,(0.922)^t$$

$$\frac{60\,000}{18\,000} = (0.922)^t$$

$$\frac{1}{3} = 0.922^t$$

$t = \log_{(1-i)}\left(\frac{F}{P}\right)$

$$t = \log_{0.922}\left(\tfrac{1}{3}\right) = 13.5 \text{ years}$$

$\Rightarrow$ after 14 years it will be worth less than €60 000

8. A computer is bought for €2500.
Compare the trade-in value of the computer after 4 years based on
(a) a net loss in value of €550 per year or (b) a loss of 35% per year.

$P = €\,2500$
$t = 4$ years

Straight line depreciation

(a) Total loss $= €550 \times 4 = €\,2200$

$\Rightarrow F = 2500 - 2200 = €\,300$

Reducing balance depreciation

$F = P(1-i)^t$

(b) $i = 35\% \Rightarrow 1-i = 0.65$

$F = 2500(0.65)^4 = €\,446$

9. A computer system is bought for €23 500. It depreciates at a rate of 28% per annum.
Find the value of the computer after
(i) 2 years (ii) 5 years (iii) 7 years.

$F = P(1-i)^t$

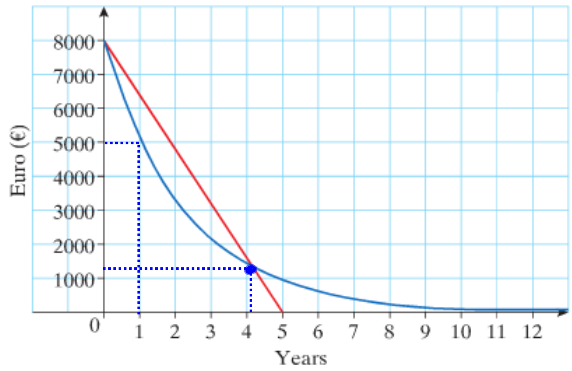
$P = €\,23500$
$i = 28\% \Rightarrow 1-i = 0.72$

$t = 2$ years (i) $F = 23500(0.72)^2 = €\,12\,182.40$

$t = 5$ years (ii) $F = 23500(0.72)^5 = €\,4\,547.06$

$t = 7$ years (iii) $F = 23500(0.72)^7 = €\,2\,357.19$

10. An air-conditioning system cost €8000. A straight line depreciation and a reducing balance curve for this system are shown below.



(i) Using the graph, estimate the rate of depreciation. (Let the value after 20 years be €1.)
(ii) Explain why the reducing balance curve can never have a zero value.
(iii) Find the slope of the straight line representing depreciation.
(iv) Estimate the point of intersection of the two graphs.
(v) After 5 years, what is the value of the system on a reducing balance basis?
(iv) In your opinion, which method of depreciation gives a more realistic value for the system? Explain your answer.

Method 1

$F=P(1-i)^t$

$\Rightarrow 1-r = \sqrt[t]{\frac{F}{P}}$

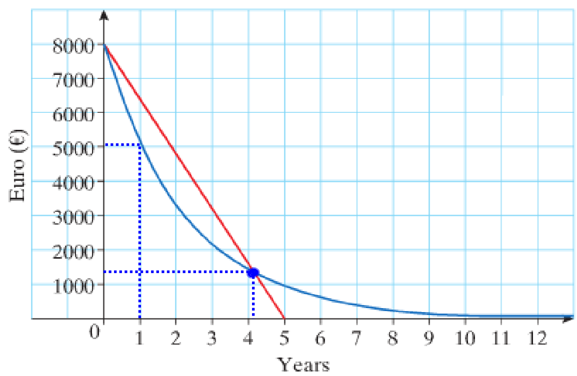
$\Rightarrow r = 1-\sqrt[t]{\frac{F}{P}}$

(i) From graph

$t=20 \quad P=€8000 \quad F=€1 \quad r=?$

$r = 1-\sqrt[20]{\frac{1}{8000}} = 0.362 = 36.2\%$

10. An air-conditioning system cost €8000. A straight line depreciation and a reducing balance curve for this system are shown below.



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Method 2

(i) From graph €8000 depreciates to €5000 in 1 year

$F=P(1-i)^t$

$t=1$

$F=5000$

$P=8000$

$5000 = 8000(1-i)^1$

$1-i = \frac{5000}{8000} = 0.625$

$\Rightarrow i = 1-0.625 = 0.375 = 37.5\%$

Q10

(ii) Because the function approaches a limit.

The value is always being reduced by 36%, but a positive number being multiplied by 0.64 is always > 0.

(iii) $|m| = \frac{\text{Rise}}{\text{Run}} = \frac{8000}{5} = 1600$

Slope is negative

$\Rightarrow m = -1600$

(iv) From graph intersection point $\approx (4 \cdot 2, 1300)$

Reducing balance Value in 5 years?

(v) $P = 8000$

$i = 36.2\%$ (using estimate part i)

$t = 5$

$F = P(1+i)^t$

$F = 8000\,(0 \cdot 638)^5 = €\,845 \cdot 66$

(v) The reducing balance is more realistic because the AC System should still have a value in 5 years.