(a) Donagh is arranging a loan and is examining two different repayment options.

(i) Bank A will charge him a monthly interest rate of 0·35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·35%.

$$F = P(1+i)^t = 1(1+0\cdot 0035)^{12} = 1\cdot 042818$$
$$\Rightarrow i = 4\cdot 28\,\%$$

(ii) Bank B will charge him a rate that is equivalent to an APR of 4·5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4·5%.

$$F = P(1+i)^t$$
$$1\cdot 045 = 1(1+i)^{12}$$
$$1+i = \sqrt[12]{1\cdot 045} = 1\cdot 0036748$$
$$\Rightarrow i = 0\cdot 367\,\%$$

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$$A = P\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$
$$= 80000\left[\frac{0\cdot0035(1\cdot0035)^{120}}{(1\cdot0035)^{120} - 1}\right]$$
$$= 80000\left[\frac{0\cdot00532296}{0\cdot520846}\right]$$
$$= 817\cdot59 = €818$$

or

$$80000 = \frac{A}{1\cdot0035} + \frac{A}{1\cdot0035^2} + \ldots + \frac{A}{1\cdot0035^{120}}$$
$$= A\left[\frac{1}{1\cdot0035} + \frac{1}{1\cdot0035^2} + \ldots + \frac{1}{1\cdot0035^{120}}\right]$$
$$= A\left[\frac{\frac{1}{1\cdot0035}\left(1 - \left(\frac{1}{1\cdot0035}\right)^{120}\right)}{1 - \frac{1}{1\cdot0035}}\right]$$
$$= A\left[\frac{0\cdot342471198}{0.0035}\right]$$
$$= A\left[97\cdot8489137\right]$$

$$A = 817\cdot58 = €818$$

We have

$$P = \frac{F}{1+i} = \frac{20000}{1.03} = 19417.48$$

So the present value is €19417.48 to the nearest cent.

(b) Write down, in terms of t, the present value of a future payment of €20,000 in t years' time.

We have

$$P = \frac{F}{(1+i)^t} = \frac{20000}{(1.03)^t}.$$

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the final fund required.

Using the solution to part (b), we see that the amount required on the date of requirement is given by

$$A = 20000 + \frac{20000}{1.03} + \cdots \frac{20000}{(1.03)^{24}}$$

Using the notation of the formula on page 22 of the Formula and Tables book, we have a geometric series with $a = 20000$, $r = \frac{1}{1.03}$ and $n = 25$. Therefore

$$A = \frac{20000\left(1 - \left(\frac{1}{1.03}\right)^{25}\right)}{1 - \frac{1}{1.03}}$$

Using a calculator we obtain

$$A = €358711$$

to the nearest euro.

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant places, the rate of interest per month that would, if paid and compounded annually, be equivalent to an effective annual rate of 3%.

We must solve $(1+i)^{12} = 1.03$. So $(1+i) = \sqrt[12]{1.03}$. Therefore

$$i = \sqrt[12]{1.03} - 1 = 0.002466$$

correct to 4 significant places. So the answer is

$$0.2466\%.$$

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(ii) Write dowm, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

Using the formula on page 30 of the Formula and Tables booklet we obtain

$$P(1.002466)^n.$$

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(iii) If Pádraig makes 480 equal payments of €P from now until his retirement, what value of P will give him the fund he requires?

We must solve

$$P(1.002466)^{480} + P(1.002466)^{479} + \cdots + P(1.002466) = 358711$$

or

$$P(1.002466 + (1.002466)^2 + \cdots + (1.002466)^{480}) = 358711$$

Using the formula for the sum of a geometric series, we obtain

$$P\left(\frac{1.002466(1-(1.002466)^{480})}{1-1.002466}\right) = 358711$$

or

$$P(919.38) = 358711.$$

Therefore $P = \frac{358711}{919.38} =$ €390.17 to the nearest cent.

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(e) If Pádraig waits for ten years before starting his pension fund, how much will he then have to pay each month in order to generate the same pension fund?

Now the number of months until his retirement date is $30 \times 12 = 360$. So as above we must solve

$$P(1.002466)^{360} + P(1.002466)^{359} + \cdots + P(1.002466) = 358711$$

or

$$P(1.002466 + (1.002466)^2 + \cdots + (1.002466)^{360}) = 358711$$

Using the formula for the sum of a geometric series, we obtain

$$P\left(\frac{1.002466(1-(1.002466)^{360})}{1-1.002466}\right) = 358711$$

or

$$P(580.11) = 358711.$$

Therefore, in this case, $P = \frac{358711}{580.11} =$ €618.35 to the nearest cent.

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