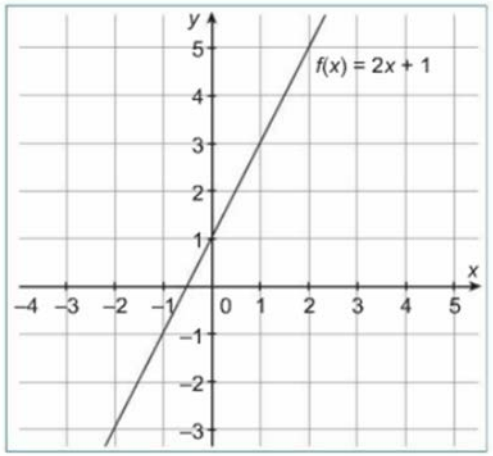
Functions and Graphs

The co-ordinates for graphs can be found by using the 'Table' mode on your calculator.

The **domain** is the x values entered into the function and the **range** the y values that come out.

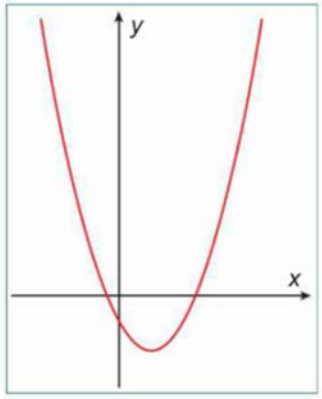
A **function** is a rule which changes one number into another. Questions involving functions involve replacing the x variable with numbers.

Linear

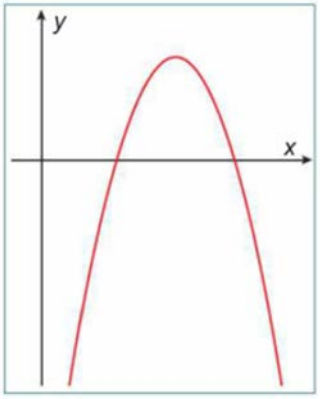


A **linear** function $f(x) = 2x + 1$ will give you a straight line.
It will have no turning point

Quadratic



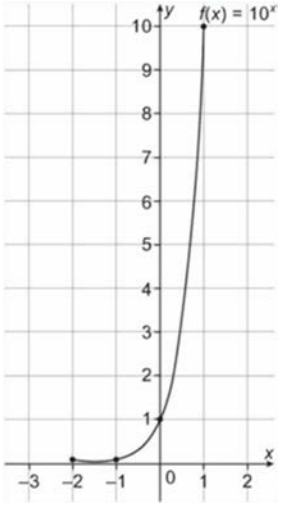
$y = ax^2 + bx + c$
Here, a is **positive**.



$y = ax^2 + bx + c$
Here, a is **negative**.

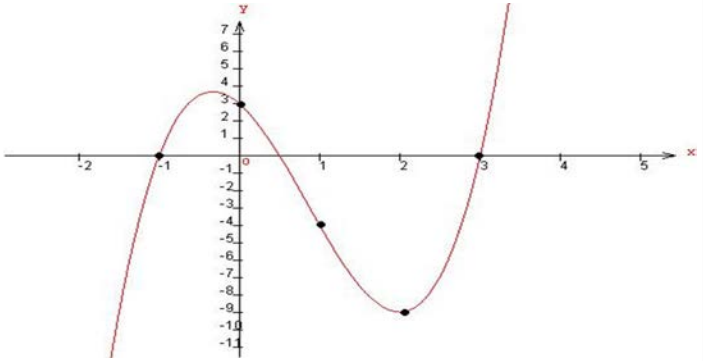
A **quadratic** function $f(x) = 3x^2 + 5x + 4$ will give a curve with only one turning point.

Exponential



An **exponential** function contains the x in the power.

Cubic



A **cubic** function $f(x) = 2x^3 - 5x^2 - 4x + 3$ will give a curve with more than one turning point.

Differentiation and Graphs

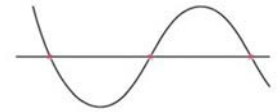
$\frac{dy}{dx} = slope$ of the tangent to a curve at any point on the curve
We can then use this slope and a point to find the **equation of a tangent**

$\frac{dy}{dx}$ can be used to determine if slope **increasing** or **decreasing**.
Positive = increasing Negative=decreasing

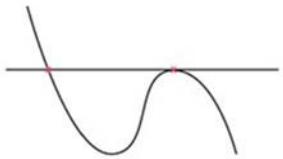
To find the **max** and **min** points of a curve we find $\frac{dy}{dx}$ and let equal to 0.

To find the **points of inflection** we find $\frac{d^2y}{dx^2}$ and let equal to 0.

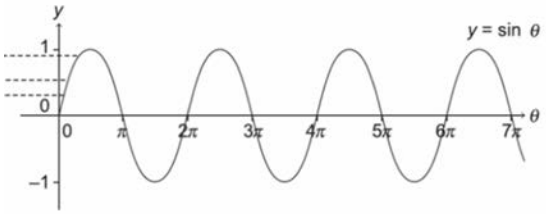
The **roots** of a function are the values of x where the curve/ line crosses the x-axis.



3 real + different roots



Graphing the Sine Function ($y = \sin\theta$)

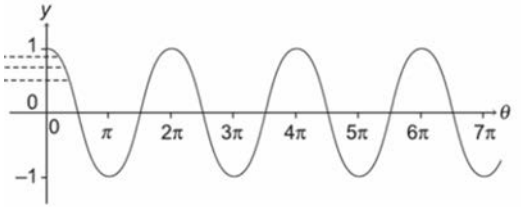


θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$y = \sin\theta$	0	0.7	1	0.7	0	−0.7	−1	−0.7	0

- $\sin 0° = 0$, so the curve passes through the origin.
- The maximum value of $\sin\theta$ is 1.
- The minimum value of $\sin\theta$ is −1.
- The graph repeats itself every 2π radians, so it is a **periodic function**.

Period = 2π
Range = [−1,1]

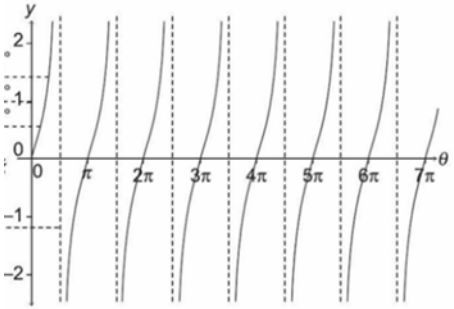
Graphing the Cosine Function ($y = \cos\theta$)



- $\cos 0° = 1$ so, the curve cuts the y-axis at (0,1).
- The maximum value of $\cos\theta$ is 1.
- The minimum value of $\cos\theta$ is −1.
- The graph repeats itself every 2π radians, so it is a **periodic function**.

Period = 2π
Range = [−1,1]

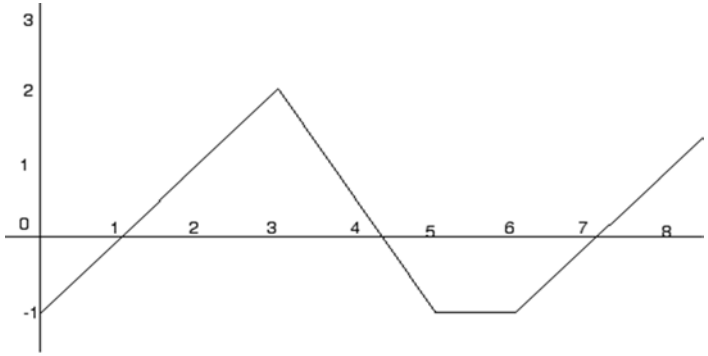
Graphing the Tangent Function ($y = \tan\theta$)



- $\tan 0° = 0$ so, the curve passes through the origin (0,0).
- There are no maximum and minimum values.
- The graph repeats itself every π radians, so it is a **periodic function**.
- There are asymptotes at $\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$ [θ given in radians] (at all odd multiples of $\frac{\pi}{2}$ radians).

Period = π

Period and Range

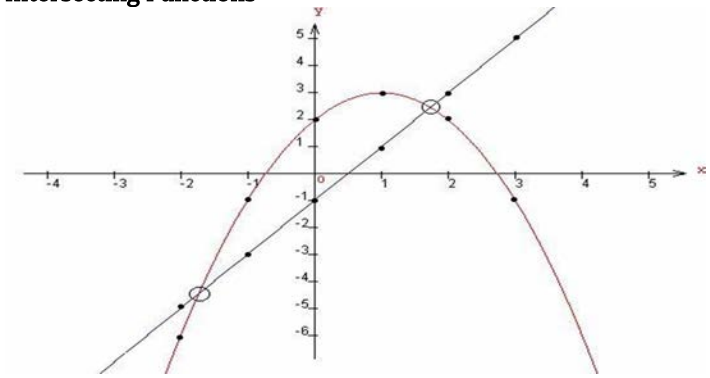


The **period** is the width of the pattern $= 6$ units
The **range** is the highest and lowest y value $[-1, 2]$

Plotting trigonometric functions

$f(x) = a\sin nx$
$g(x) = a\cos nx$
$Range\ [-a, a]\ Period = \frac{2\pi}{n}$

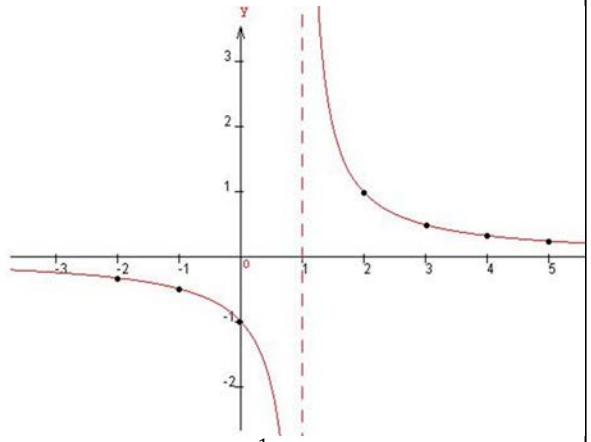
Intersecting Functions



If $f(x) = g(x)$ we can solve for x by drawing the two functions and identifying the x values on the graph of the points where the curves/ lines meets.

We can do it algebraically either with a simultaneous equation.

Asymptote



An asymptote $f(x) = \frac{1}{x-1}$ will give two curves on the same graph that don't intersect.

They have no turning points or points of inflection.
They are always increasing or decreasing

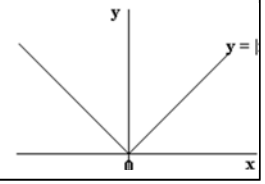
To Sketch Asymptotes:

Vertical Asymptote: Bottom $= 0$
Horizontal Asymptote $y = \lim_{x\to\infty} f(x)$

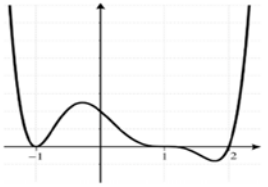
Vertical Asymptote
$x - 1 = 0$
$x = 1$

Horizontal Asymptote
$\lim_{x\to\infty} \frac{1}{x-1} = 0$
$y = 0$

Modular Functions



Graphing Polynomials



Max number of roots polynomial can have is its highest power.
Where curve touches axis but doesn't cross is repeated root.