Example 1

From the given number of names and surnames, determine how many different name–surname pairs are possible.

There are 6 names and for each of these, there are 5 surnames.

Name	Surname
Mary	Mooney
Jennifer	Byrne
Cormac	O'Brien
Kate	Lawiski
Barry	
Shane	McCarthy

Fundamental Principle of Counting

$6 \times 5 = 30$ ✓

Example 2

There are 6 different books, including a science book, on a shelf.

(i) In how many different ways can the 6 books be arranged on the shelf?

(ii) In how many ways can the 6 books be arranged if the science book is always on the extreme left?

(i) $6! = 720$ ✓

(ii) $(5!)(1) = 120$ ✓

Example 3

In how many ways can the letters of the word SCOTLAND be arranged in a line?
(i) In how many of these arrangements do the two vowels come together?
(ii) How many of the arrangements begin with S and end with the two vowels?

SCOTLAND has 8 letters

arrangements = 8! = 40,320 ✓

(i) Treat vowels as 1 letter

find arrangements of 7 letters = 7!
2 vowels can be in 2! or 2 arrangements

ANSWER = (7!)(2) = 10,800 ✓

(ii) Begin with S, 5 letters then end with vowels

$\boxed{1} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{\boxed{2} \times \boxed{1}}$

$= (5!)(2) = 240$ ✓

Example 4

How many four-digit numbers can be formed using the digits $\underbrace{0, 2, 5, 7, 8}_{5 \text{ digits}}$ if a digit cannot be used more than once in any number?
(i) How many of these numbers are greater than 5000?
(ii) How many of these numbers are odd?

0 can't be the first digit!

4 digit nos = $\boxed{4} \times \boxed{4} \times \boxed{3} \times \boxed{2} = 96$ ✓

(i) greater than 5000 ⇒ first digit is 5, 7 or 8.

$\boxed{3} \times \boxed{4} \times \boxed{3} \times \boxed{2} = 72$ ✓

(ii) greater than 5000 and odd

odd ⇒ ends in 5 or 7.

$\boxed{3} \times \boxed{3} \times \boxed{2} \times \boxed{2} = 36$ ✓

Example 5

(i) Evaluate ${}^{10}P_3$ (ii) Find n if $7[{}^nP_3] = 6[{}^{n+1}P_3]$

(i) ${}^{10}P_3 = 720$ ✓

(ii) Find n?

$$7[{}^nP_3] = 6[{}^{(n+1)}P_3]$$

${}^nP_r = (n+1)\dots(n-(r-1))$

$$\Rightarrow 7[\cancel{n}\,\cancel{(n-1)}(n-2)] = 6[(n+1)\cancel{(n)}\cancel{(n-1)}]$$

$$7(n-2) = 6(n+1)$$

$$7n - 14 = 6n + 6$$

$$n = 20 \checkmark$$

${}^nP_r = \dfrac{n!}{(n-r)!}$

or $\Rightarrow 7\left[\dfrac{n!}{(n-3)!}\right] = 6\left[\dfrac{(n+1)!}{(n+1-3)!}\right]$

$$\Rightarrow 7\left[\frac{\cancel{n!}}{(n-3)!}\right] = 6\left[\frac{\cancel{n!}\,(n+1)}{(n-2)!}\right]$$

$$\Rightarrow 7/\cancel{(n-3)!} = 6(n+1)/\cancel{(n-3)!}\,(n-2)$$

$$\Rightarrow 7(n-2) = 6(n+1) \Rightarrow n = 20$$

Example 6

How many different four-letter arrangements can be made from the letters of the word THURSDAY if a letter cannot be repeated in an arrangement?
How many of the arrangements begin with the letter D and end with a vowel?

8 letters in Thursday

(i) $\boxed{8} \times \boxed{7} \times \boxed{6} \times \boxed{5} = 1{,}680$ ✓

(ii) $\boxed{1} \times \boxed{6} \times \boxed{5} \times \boxed{2} = 60$ ✓

Example 1

(i) In how many ways can a team of 5 players be chosen from 9 players?
(ii) In how many ways can this be done if a certain player must be selected in each team?

(i) $\binom{9}{5} = 126$ ✓

(ii) If one player is already selected then we are only choosing 4 from 8.

$\binom{8}{4} = 70$ ✓

Example 2

(i) In how many ways can a group of five be selected from ten people?
(ii) How many groups can be selected if two particular people from the ten cannot be in the same group?

(i) $\binom{10}{5} = 252$ ✓

(ii) 2 people can't be in the same group?

Ways that 2 people are always selected
$\Rightarrow$ now choosing 3 from 8

$^8C_3 = 56$

All other ways these 2 people are separated
$\Rightarrow$ ANSWER $= 252 - 56 = 196$ ✓

Example 3

Find the number of ways in which a panel of four men and three women can be chosen from seven men and five women.

7 men 'and' 5 women
choose 4 chose 3

$$\binom{7}{4} \times \binom{5}{3}$$

$$(35)(10) = 350 \checkmark$$

Example 4

In how many ways can a committee of six be formed from 5 teachers and 8 students if there are to be more teachers than students on each committee?

more teachers than students
⇒ exactly 4 teachers and 2 students
'or' 5 teachers and 1 student

$$= \binom{5}{4} \times \binom{8}{2} + \binom{5}{5}\binom{8}{1}$$

$$= (5)(28) + (1)(8) = 148 \checkmark$$

Example 1

If a card is drawn from a pack of 52, find the probability that it is
(i) an ace (ii) a diamond (iii) a red card.

(i) 1 Ace in every suit

$P(\text{ace}) = \frac{1}{13}$ ✓

(ii) $\frac{1}{4}$ cards are diamond

$P(\text{diamond}) = \frac{1}{4}$ ✓

(iii) $\frac{1}{2}$ cards are red

$P(\text{red}) = \frac{1}{2}$ ✓

Example 2

A letter is selected at random from the letters of the word $\underbrace{\text{STATISTICS}}_{\text{10 letters}}$.
Find the probability that the letter is
(i) C (ii) S (iii) S or T (iv) a vowel.

(i) $P(C) = \frac{1}{10}$ ✓

(ii) $P(S) = \frac{3}{10}$ ✓

(iii) $P(S \text{ or } T) = \frac{6}{10} = \frac{3}{5}$ ✓

or $P(S \text{ or } T) = P(S) + P(T) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$

(iv) $P(\text{Vowel}) = 3/10$ ✓

Example 3

If two dice are thrown and the scores are added, set out a sample space giving all the possible outcomes. Find the probability that

(i) the total is exactly 7 (ii) the total is 4 or less

(iii) the total is 11 or more (iv) the total is a multiple of 5.

SAMPLE SPACE

dice 1

dice 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(i) $P(7) = 6/36 = 1/6$ ✓

(ii) $P(4 \text{ or less}) = 6/36 = 1/6$ ✓

(iii) $P(11 \text{ or more}) = 3/36 = 1/12$ ✓

multiples of 5 are 5, 10

(iv) $P(\text{multiple of } 5) = 7/36$ ✓

Example 1

Dara collected data on the colours of cars passing the school gate.
His results are shown on the table below.

Colour	White	Red	Black	Blue	Green	Other
Frequency	24	32	14	16	10	4

(i) How many cars did Dara survey?
(ii) What was the relative frequency of blue cars?
(iii) What was the relative frequency of red cars?
Give your answer as a decimal.
(iv) Write down an estimate of the probability that the next car passing the school gate will be green.
(v) How can the estimate for the probability of green cars be made more reliable?

(i) 100 ✓

(ii) Relative frequency blue $= 16/100 = 4/25$ ✓

(iii) Relative frequency red $= 32/100 = 0{\cdot}32$ ✓

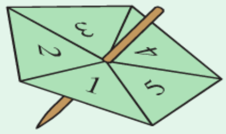
Relative frequency = Estimate of probability

(iv) $P(\text{green}) = 10/100 = 1/10$ ✓

(v) Do a larger survey ✓

Example 2

This spinner is biased.
The probability that the spinner will land on each of the numbers 1 to 4 is given in the table below.



Number	1	2	3	4	5
Probability	0.35	0.1	0.25	0.15	k

The spinner is spun once.
(i) Work out the probability, k, that the spinner will land on 5.
(ii) Write down the number on which the spinner is most likely to land.
(iii) If the spinner is spun 200 times, how many times would you expect it to land on 3?

Sum of probabilities = 1

(i) $k = 1 - (0.35 + 0.1 + 0.25 + 0.15)$
$k = 0.15$ ✓

(ii) $P(1) = 0.35$ is greatest ✓

Expected Value = Probability x no. of trials

(iii) expected 3's $= (0.25)(200) = 50$ ✓

Example 1

A card is drawn at random from a pack of 52.
What is the probability that the card is
(i) a club (ii) a king (iii) a club or a king
(iv) a red card (v) a queen (vi) a red card or a queen

(i) $P(\text{club}) = \frac{1}{4}$ ✓

(ii) $P(\text{King}) = \frac{1}{13}$ ✓

(iii) $P(\text{club or King}) = P(\text{club}) + P(\text{King}) - P(\text{Both})$
$= \left(\frac{1}{4}\right) + \left(\frac{1}{13}\right) - \left(\frac{1}{52}\right) = \frac{4}{13}$ ✓

(iv) $P(\text{Red}) = \frac{1}{2}$ ✓

(v) $P(\text{queen}) = \frac{1}{13}$ ✓

(vi) $P(\text{Red or queen}) = P(\text{red}) + P(\text{queen}) - P(\text{both})$
$= \left(\frac{1}{2}\right) + \left(\frac{1}{13}\right) - \left(\frac{2}{52}\right) = \frac{7}{13}$ ✓

Example 2

A and B are two events such that $P(A) = \frac{19}{30}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{4}{5}$. Find $P(A \cap B)$.

general addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{19}{30} + \frac{2}{5} - \frac{4}{5}$$

$$= \frac{7}{30} \checkmark$$

Example 3

The given Venn diagram represents the subjects taken by a group of 50 pupils.

(i) Find the value of x.

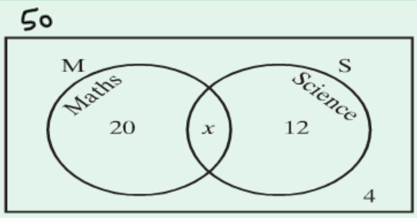
Now find the probability that a person chosen at random takes

(ii) both subjects

(iii) neither subject

(iv) Science but not Maths

(v) Maths or Science or both.



(i) $20 + x + 12 + 4 = 50$

$36 + x = 50$

$x = 50 - 36 = 14 \checkmark$

(ii) $P(M \cap S) = \frac{14}{50} = \frac{7}{25} \checkmark$

(iii) $P(\text{neither}) = \frac{4}{50} = \frac{2}{25} \checkmark$

(iv) $P(S \setminus M) = \frac{12}{50} = \frac{6}{25} \checkmark$

(v) $P(M \cap S) = \frac{(20+14+12)}{50} = 46/50 = 23/25 \checkmark$

or $P(M \cap S) = 1 - P(\text{neither}) = 1 - \frac{2}{25} = 23/25$

Example 1

When two dice are thrown, what is the probability of getting
(i) two sixes (ii) 4 or more on each die?

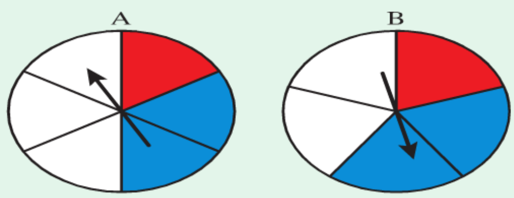
(i) $P(6,6) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$ ✓

4,5 or 6 are ≥ 4

(ii) $P(\geq 4, \geq 4) = \left(\frac{3}{6}\right)\left(\frac{3}{6}\right) = \frac{1}{4}$ ✓

Example 2

These two spinners are spun.
What is the probability that
(i) spinner A shows red
(ii) spinner B shows red
(iii) both spinners show red
(iv) A shows red and B shows blue
(v) both show blue
(vi) both show white
(vii) neither shows white?



(i) Spinner A, $P(\text{Red}) = \frac{1}{6}$ ✓

(ii) Spinner B, $P(\text{Red}) = \frac{1}{5}$ ✓

(iii) $P(\text{Red}, \text{Red}) = \left(\frac{1}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{30}$ ✓

(iv) $P(\overset{A}{\text{Red}}, \overset{B}{\text{blue}}) = \left(\frac{1}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{15}$ ✓

(v) $P(\text{blue} . \text{blue}) = \left(\frac{2}{6}\right)\left(\frac{2}{5}\right) = \frac{2}{15}$ ✓

(vi) $P(\text{white}, \text{white}) = \left(\frac{3}{6}\right)\left(\frac{2}{5}\right) = \frac{1}{5}$ ✓

(vii) $P(\text{not white}, \text{not white}) = \left(\frac{3}{6}\right)\left(\frac{3}{5}\right) = \frac{3}{10}$ ✓

Example 3

A gambler must throw a 6 with a single dice to win a prize.
Find the probability that he wins at his third attempt.

(S) Success = 6 (F) fail = not 6

$P(S) = 1/6$ $P(F) = 5/6$

$P(F, F, S) = (5/6)(5/6)(1/6) = 25/216$ ✓

Example 4

Three pupils A, B and C have their birthdays in the same week.
What is the probability that the three birthdays
(i) fall on a Monday
(ii) fall on the same day
(iii) fall on three different days?

(i) $P(\overset{A}{\text{monday}}, \overset{B}{\text{monday}}, \overset{C}{\text{monday}}) = (\frac{1}{7})(\frac{1}{7})(\frac{1}{7}) = \frac{1}{343}$ ✓

(ii) $P(\text{same day}) = P(\text{particular day}) \times 7 = \frac{7}{343} = \frac{1}{49}$ ✓

(iii) $P(3 \text{ different days}) = ?$

$= P(\text{A born any day, B born different day, C born separate day})$
$= (\frac{7}{7})(\frac{6}{7})(\frac{5}{7}) = 30/49$ ✓

Example 1

The numbers 1 to 9 are written on cards and placed in a box.
A card is drawn at random from the box.
Find the probability that the number is prime, given that the number is odd.

Conditional probability

odd numbers are 1, 3, 5, 7, 9
of these 3, 5 and 7 are prime

$\Rightarrow P(\text{prime} \mid \text{odd}) = 3/5$ ✓

or

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

2, 3, 5, 7 are prime
$\Rightarrow \frac{3}{9}$ are prime and odd

$$P(\text{prime} \mid \text{odd}) = \frac{P(\text{prime and odd})}{P(\text{odd})}$$

$$= (3/9)/(5/9)$$

$$= 3/5$$

Example 2

A bag contains 6 red and 4 blue discs. A disc is drawn from the bag and not replaced. A second disc is then drawn.
Find the probability that
(i) the first two discs are blue
(ii) the second disc drawn is red
(iii) one disc is red and the other disc is blue
(iv) both discs are the same colour.

(i) $P(\text{1st is blue}) = 4/10 = 2/5$
$P(\text{2nd is blue} \mid \text{1st is blue}) = 3/9$
$\Rightarrow P(\text{Blue, Blue}) = (2/5)(3/9) = 2/15$ ✓

(ii) $P(\text{2nd disc is Red})$?

⇒ either 1st disc is Red and 2nd disc is red
or 1st disc is blue and 2nd disc is red

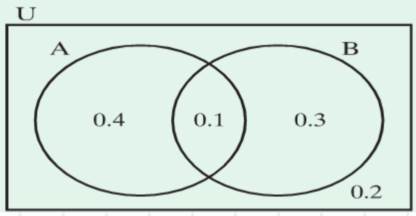
$P(\text{Red, Red}) = (6/10)(5/9) = 1/3$
$P(\text{Blue, Red}) = (4/10)(6/9) = 4/15$
$P(\text{Red, Red} \cup \text{Blue, Red}) = 1/3 + 4/15 = 3/5$ ✓

(iii) $P(R,B \cup B,R) = (6/10)(4/9) + (4/10)(6/9) = 8/15$ ✓

(iv) $P(\text{both same}) = P(RR \cup B,B) = \frac{1}{3} + \frac{2}{15} = 7/15$ ✓

Example 3

Use the given Venn diagram to write down
(i) $P(A|B)$ (ii) $P(B|A)$



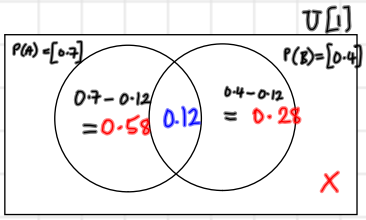
(i) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0.1}{0.4} = 0.25$

(ii) $P(B|A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{0.1}{0.5} = 0.2$

Example 4

Two events A and B are such that $P(A) = 0.7$, $P(B) = 0.4$ and $P(A|B) = 0.3$.
Determine the probability that neither A nor B occurs.

Draw Venn diagram



$X = P(\text{neither occurs})$

$P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A\cap B) = P(A|B) \times P(B) = (0.3)(0.4) = 0.12$

$\Rightarrow P(\text{neither}) = 1 - (0.58 + 0.12 + 0.28) = 0.02$