

2006 Q8 b(i)

Substitute

$$\text{Integrate } \int_1^2 x(1+x^2)^3 dx = \int_1^2 (1+x^2)^3 x dx$$

$$\text{let } u = 1 + x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

change limits

$$u = 1 + (2)^2 = 5$$

$$u = 1 + (1)^2 = 2$$

Rewrite in terms of u

$$\int_2^5 u^3 \left(\frac{1}{2} du\right) = \frac{1}{2} \int_2^5 u^3 du$$

Integrate

$$= \frac{1}{2} \left[ \frac{u^4}{4} \right]_2^5$$

evaluate

$$= \frac{1}{2} \left[ \frac{(5)^4}{4} - \frac{(2)^4}{4} \right] = \frac{1}{2} \left[ \frac{625-16}{4} \right] = \frac{1}{2} \left[ \frac{609}{4} \right]$$

$$= \frac{609}{8}$$