(a) Find $\int (x^3 + \sqrt{x})\, dx$.

$\div \frac{3}{2} = \times \frac{2}{3}$

$$= \frac{x^4}{4} + \frac{2x^{\frac{3}{2}}}{3} + c$$

(b) (i) Evaluate $\int_0^2 \frac{x+1}{x^2+2x+2}\, dx$. $= \int \left(\frac{1}{x^2+2x+2}\right)(x+1)\, dx$

let $u = x^2+2x+2$

$\Rightarrow$ $\frac{du}{dx} = 2x+2$

$du = (2x+2)\, dx$

$\frac{1}{2} du = (x+1)\, dx$

Change limits

$u_1 = (2)^2 + 2(2) + 2 = 10$

$u_2 = (0)^2 + 2(0) + 2 = 2$

Rewrite integral in terms of u

$$= \frac{1}{2} \int_2^{10} \frac{1}{u}\, du = \frac{1}{2}\Big[\ln u\Big]_2^{10}$$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

$$= \frac{1}{2}\Big[\ln 10 - \ln 2\Big] = \frac{1}{2} \ln \frac{10}{2} = \frac{\ln 5}{2}$$

(ii) Evaluate $\int_0^2 \frac{x^2+2x+2}{(x+1)} dx$.

$u = x+1$ $\quad x = (u-1)$

$\frac{du}{dx} = 1$

$du = dx$

$\Rightarrow x^2+2x+2$

$= (u-1)^2 + 2(u-1) + 2$

$= u^2 - \cancel{2u} + 1 + \cancel{2u} - \cancel{2} + \cancel{2}$

$= u^2+1$

limits

$2 \rightarrow 3$

$0 \rightarrow 1$

Rewrite integral in terms of u

$\int_1^3 \frac{1}{u}(u^2+1)\,du \quad \int_1^3 \left(u + \frac{1}{u}\right) du$

$= \left[\frac{u^2}{2} + \ln u\right]_1^3 = \left[\left(\frac{(3)^2}{2} + \ln 3\right) - \left(\frac{(1)^2}{2} + \ln 1\right)\right]$

$= \frac{9}{2} + \ln 3 - \frac{1}{2} - \ln 1$

$= 4 + \ln 3$ ✓

note:

$\ln 3 - \ln 1$

$= \ln \frac{3}{1} = \ln 3$