HIGHER COURSE: TOPICS

Note for the examination:

In the case of results marked with an asterisk (*), formal proofs may be examined; in the case of other results stated in the syllabus, proofs will not be examined.

Functions and calculus

1. Functions:

Finding the period and range of a continuous periodic function, given its graph on scaled and labeled axis.

Informal treatment of limits of functions; rules for sums, products and quotients.

2. Differential calculus:

*Derivations from first principles of x^2 , x^3 , $\sin x$, $\cos x$, \sqrt{x} , and 1/x.

First derivatives of:

- polynomials, rational, power and trigonometric functions;
- tan⁻¹, sin⁻¹, exponential and logarithmic functions;
- *sums; *products; differences; *quotients; compositions of these

*Proof by induction that
$$\frac{d}{dx}(x^n) = n x^{n-1}$$
.

Application to finding tangents to curves.

Simple second derivatives.

First derivatives of implicit and parametric functions.

Rates of change.

Maxima and minima.

Curve sketching of polynomials and of functions of the form

$$\frac{a}{x+b}$$
 and $\frac{x}{x+b}$, with reference to turning points, points of inflections,

and asymptotes.

Newton-Raphson method for finding approximate roots of cubic equations.

Range a closed interval [a, b], $a, b \in \mathbb{Z}$; period $\in \mathbb{N}_0$.

Periodic graph need not necessarily be trigonometric in type: e.g. saw-tooth graph.

Problems involving modelling excluded.

3. Integral calculus:

Integration techniques (integrals of sums, multiplying constants, and substitution) applied to:

- a. x^n
- b. $\sin nx$, $\cos nx$, $\sin^2 nx$, $\cos^2 nx$;
- c. e^{nx}
- d. functions of the form:

$$\frac{1}{x+a}$$
, $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{(a^2-x^2)}}$, $\sqrt{(a^2-x^2)}$.

Definite integrals with applications to areas and volumes of revolution (confined to cones and spheres).

Integration by parts and partial fractions <u>excluded</u>.