Section B	**Contexts and Applications**	**150 marks**

Answer Question 7 and Question 8.

Question 7 **(75 marks)**

(a) Some students are using a database of earthquakes to investigate the times between the occurrences of serious earthquakes around the world. They extract information about all of the earthquakes in the 20th century that caused at least 1000 deaths. There are 115 of these.

The students wonder whether there are patterns in the timing of these earthquakes, so they look at the number of days between each successive pair of these earthquakes.

They make the following table, showing the number of earthquakes for which the time interval from the previous earthquake is as shown.

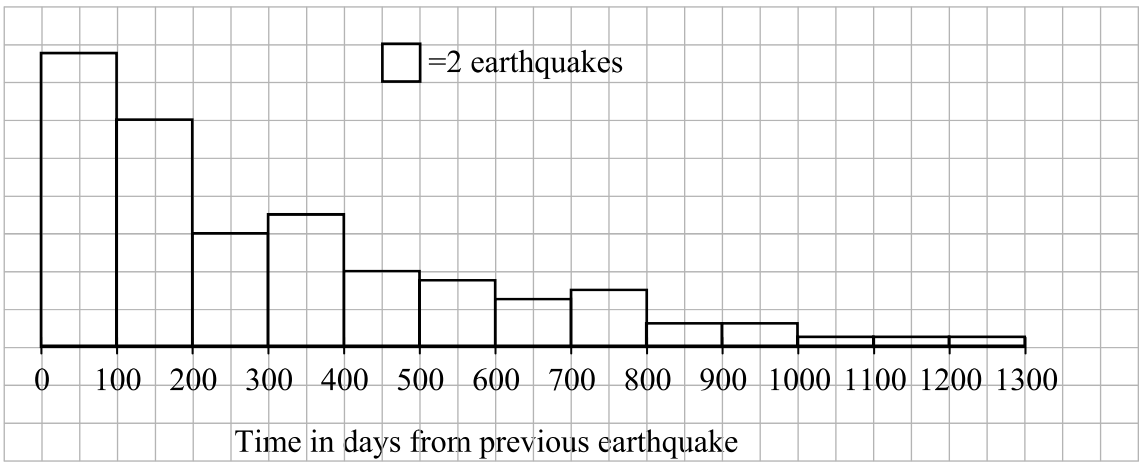
Time in days from previous earthquake	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 1000	1000 – 1300
Number of earthquakes	31	24	12	14	8	7	5	6	5	3

[Source: National geophysical data center, significant earthquake database: www.ngdc.noaa.gov]

(i) Create a suitable graphical representation of the distribution.

Histogram.
First, divide up unequal intervals, and estimate an allocation of the data (optional step).

800 – 900	900 – 1000	1000 – 1100	1100 – 1200	1200 – 1300
2·5	2·5	1	1	1



(ii) Describe the distribution. Your description should refer to the shape of the distribution and should include an estimate of the median.

> The distribution is skewed to the right. (Or, e.g., there is a lot of data to the left, and it tails off to the right, etc.)
>
> The median is approximately 220 days.

(iii) The mean time between these earthquakes is 309 days and the standard deviation is 277 days. Suppose that such an earthquake has just occurred and that we want to find the probability that the time to the next one will be between 100 and 200 days. Explain why it would **not** be correct to use standard normal distribution tables (z-tables) to do this.

> Because the distribution is not normal.

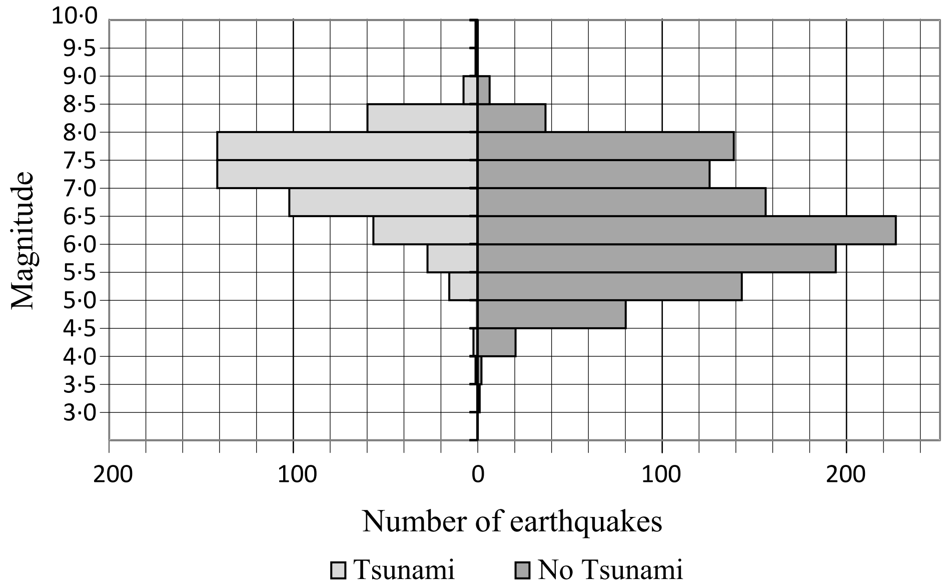
(iv) Based on the information presented in this question so far, what is the best estimate for the probability described in part **(iii)** above? Explain your reasoning.

> The best estimate is to assume the probability is as reflected in the proportion of such intervals in the historical data.
>
> $$\frac{24}{115} \approx 0{\cdot}2$$

(v) As stated at the beginning, the students chose to analyse earthquake timings by looking at the time intervals between the occurrences of a particular type of earthquake. Suggest a different way that they could have looked at the data in the database in order to try to find patterns in the timing of earthquakes.

- They could have looked at the number of earthquakes each year, or some other interval of time (e.g. distribution of earthquakes per decade, per year, etc.)
- They could have redefined serious earthquakes as earthquakes greater than a certain magnitude; earthquakes in less populated areas are not included.
- The data set could have been broadened to include less serious earthquakes. This could result in a different pattern.

(b) The students heard a reporter saying that "strong earthquakes will cause large destructive ocean waves called tsunamis, while weaker ones will not". They decide to check this. They draw two histograms back to back, one showing the magnitudes of the earthquakes that caused tsunamis, and the other showing the magnitudes of those that did not. They use all of the suitable data from the 20th century that were recorded in this particular database.



(i) Comment on the reporter's statement, using information from the diagram to support your answer, and suggest a more accurate statement.

The statement is too deterministic – strong earthquakes don't always cause tsunamis and weak ones sometimes do. A better statement would be "Strong earthquakes are more likely to cause tsunamis than weaker ones."

(ii) By taking suitable readings from the diagram, estimate the probability that an earthquake of magnitude between 6·5 and 7·0 will cause a tsunami.

About 103 of these did and about 156 didn't. So probability is $\frac{103}{259} \approx 0{\cdot}4$

(iii) Consider the next six earthquakes of magnitude at least 7·5. Find an estimate for the probability that at least four of them will cause a tsunami, assuming that these six events are independent of each other.

Tsunami : 142 + 60 + 8 =210

No tsunami: 139 + 36 + 7 = 182

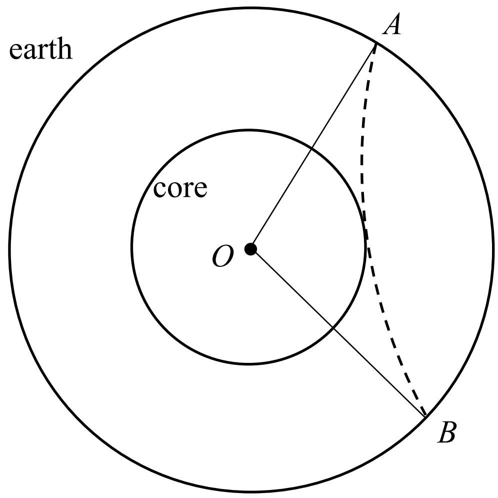
$p \approx \frac{210}{392} \approx 0{\cdot}54$ *

$$\binom{6}{4}(0{\cdot}54)^4(0{\cdot}46)^2 + \binom{6}{5}(0{\cdot}54)^5(0{\cdot}46) + (0{\cdot}54)^6 = 0{\cdot}421$$

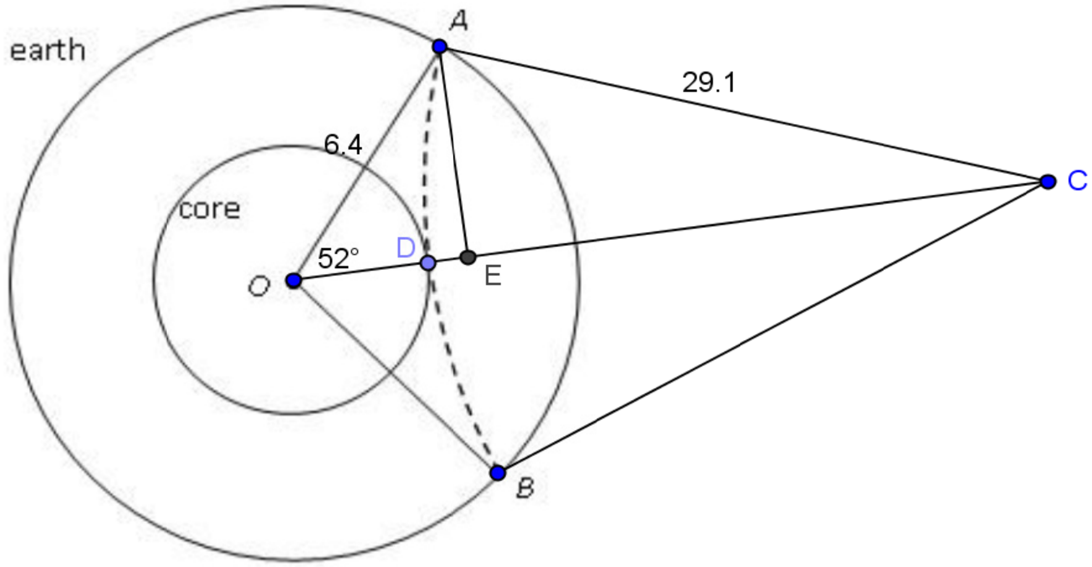
(c) Scientists use information about seismic waves from earthquakes to find out about the internal structure of the earth.

The diagram below represents a circular cross-section of the earth. The dashed curve represents the path of a seismic wave travelling through the earth from an earthquake near the surface at *A* to a monitoring station at *B*. The radius of the earth is 6·4 units and the path of this wave is a circular arc of radius 29·1 units, where 1 unit = 1000 km. Based on information from other stations, it is known that this particular path just touches the earth's core. The angle *AOB* measures 104°, where *O* is the centre of the earth.

Find the radius of the earth's core.
(There is space for work on the next page.)



* In the printed copy of the scheme, "210" incorrectly read "310". This web version has been amended to accurately reflect the scheme as used by examiners.



$|AE| = 6{\cdot}4\sin 52 = 5{\cdot}04$

$|OE| = 6{\cdot}4\cos 52 = 3{\cdot}94$

$|CE| = \sqrt{29{\cdot}1^2 - 5{\cdot}04^2} = 28{\cdot}66$

$|DE| = 29{\cdot}1 - 28{\cdot}66 = 0{\cdot}44$

$|OD| = 3{\cdot}94 - 0{\cdot}44 = 3{\cdot}5$ units (or 3500 km).

Or

Let $|OC| = x$. Then, in triangle OAC we have: $(29{\cdot}1)^2 = (6{\cdot}4)^2 + x^2 - 2(6{\cdot}4)x\cos 52°$

$x^2 - 7{\cdot}88x - 805{\cdot}85 = 0$

~~$x = -24{\cdot}7$~~ $\quad x = 32{\cdot}6$

$|OE| = 32{\cdot}6 - 29{\cdot}1 = 3{\cdot}5$ units (or 3500 km).