P 7

Solutions

Pre-Leaving Certificate Examination, 2013

Triailscrúdú na hArdteistiméireachta, 2013

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

2½ hours

300 marks

Name:	Mr. R
School:	
Address:	
Class:	
Teacher:	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7, 8 and 9.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) 20% of a consignment of oranges are known to be bad. If five oranges are selected at random, find the probability that:

(i) all five are bad.

* $P(B,B,B,B,B) = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$

$= \frac{1}{3125}$

* this solution incorrectly assumes events are independent.

(ii) at least one is bad.

* $P(\text{none bad}) = \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{1024}{3125}$

* $P(\text{at least 1 bad}) = 1 - \frac{1024}{3125} = \frac{2101}{3125}$

(b) A bag contains 8 blue marbles, 4 red marbles and x white marbles. A marble is drawn at random and not replaced. A second marble is drawn at random. If the probability that both marbles are white is $\frac{5}{51}$, how many white marbles are in the bag?

$P(w,w) = \frac{5}{51}$

$P(w) = \frac{x}{12+x}$

$P(w,w) = \left(\frac{x}{12+x}\right)\left(\frac{x-1}{11+x}\right) = \frac{5}{51}$

$\Rightarrow 51(x^2-x) = (60+5x)(11+x)$

$51x^2 - 51x = 660 + 60x + 55x + 5x^2$

$46x^2 - 166x - 660 = 0$

$23x^2 - 83x - 330 = 0$

$x = \frac{83 \pm \sqrt{83^2+4(23)(330)}}{2(23)} = 6$ ✓ or $\frac{-55}{23}$ reject

Solution: $x = 6$

Question 2 **(25 marks)**

(a) Explain what is meant by *stratified sampling* and give an example of this type of sampling.

Meaning:	The population is divided into subgroups so individuals within the subgroup share some characteristic.
Example:	Subgroups or "strata" could be gender, age group, year group...

(b) Name one measure of central tendency and give one advantage and one disadvantage of the named measure.

Measure:	Mean (or mode, median)
Advantage:	The mean is a widely understood measure of average and takes all data into account.
Disadvantage:	It can be affected by outliers.

(c) A market research company is carrying out a national poll to find out people's opinion on the involvement of the E.U. in Ireland's banking crisis. The company picks 15 towns at random from a map of Ireland. They then choose 50 phone numbers from each town at random. These people will form the sample. Discuss the validity of choosing the sample in this fashion.

The strengths in terms of validity:

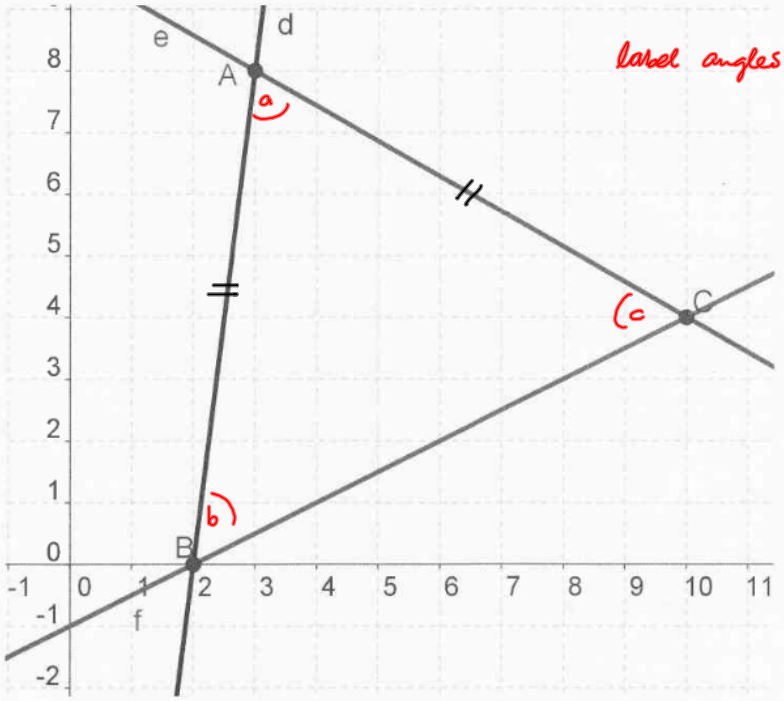
- the poll is nationwide
- the towns are randomly selected
- the poll of 750 gives a margin of Error $E = 1/\sqrt{750} = \pm 3.65\%$

Weaknesses may be:

- Only people with phone numbers included in poll.
- The survey may not be representitive as it isn't weighted - it surveys 50 from each town regardless of the size of the town.

Question 3 **(25 marks)**

(a) The lines ***d***, ***e*** and ***f*** intersect as shown. By using the formula $\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$, investigate the precise nature of the polygon formed by the lines ***d***, ***e*** and ***f***.



A(3,8) B(2,0) C(10,4)

Slopes? $m = \frac{\text{Rise}}{\text{Run}}$

$m_{BC} = \frac{4}{8} = \frac{1}{2}$, $m_{AB} = \frac{8}{1} = 8$, $m_{AC} = -\frac{4}{7}$

$a = ?$ $a = \tan^{-1}\left|\frac{8 - (-\frac{4}{7})}{1 + (8)(-\frac{4}{7})}\right| = 67.4°$

$b = ?$ $b = \tan^{-1}\left|\frac{8 - \frac{1}{2}}{1 + (8)(\frac{1}{2})}\right| = 56.3°$

$c = ?$ $c = \tan^{-1}\left|\frac{\frac{1}{2} - (-\frac{4}{7})}{1 + (\frac{1}{2})(-\frac{4}{7})}\right| = 56.3°$

⇒ It's an isosceles triangle

(b) Calculate the area of the polygon formed.

$A = (3,8)$ $B = (2,0)$

$|AB| = |AC|$ isosceles

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3-2)^2 + (8-0)^2} = \sqrt{1^2 + 8^2} = \sqrt{65}$

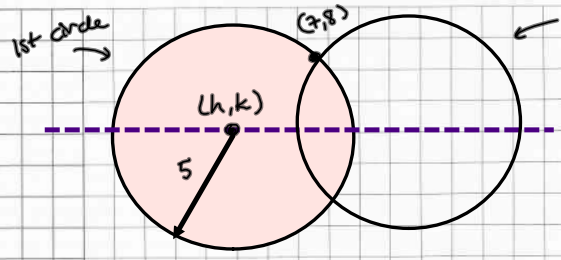
$\triangle = \frac{1}{2}\, ab \sin C = \frac{1}{2}\, |AB||AC| \sin a$

$= \frac{1}{2}\, (\sqrt{65})(\sqrt{65}) \sin 67.4°$

≈ 30 units^2

Question 4 (25 marks)

A circle of radius length 5 contains the point $(7,8)$. Its centre lies on the line $-2x+y=-4$.
Find the equations of the two circles that satisfy these conditions.



$-2x+y=-4$ (h,k) is on line

$\Rightarrow -2h+k=-4$

$\boxed{k=2h-k}$ ①

equation of circle: $(x-h)^2+(y-k)^2=5^2$

$25=(7-h)^2+(8-k)^2$

Sub in ① $\Rightarrow$ $25=(7-h)^2+(8-(2h-4))^2$

$25=(7-h)^2+(12-2h)^2$

$25=49-14h+h^2+144-48h+4h^2$

Solve $5h^2-62h+168=0$

$(5h-42)(h-4)=0$

$\Rightarrow h=\frac{42}{5}$ or $h=4$

$k=2h-4$ $\Rightarrow k=2\left(\frac{42}{5}\right)-4=\frac{64}{5}$ or 4

Centres are $\left(\frac{42}{5},\frac{64}{5}\right)$ or $(4,4)$

equations are: $\left(x-\frac{42}{5}\right)^2+\left(y-\frac{64}{5}\right)^2=25$

and $(x-4)^2+(y-4)^2=25$

Question 5 **(25 marks)**

(a) Tap A can fill a cylindrical container in 3 minutes. Tap B can fill the same container in 15 minutes. How long would it take the two taps together to fill the container?

Tap A can fill $\frac{1}{3}$ of the container in 1 min.

Tap B can fill $1/15$ of the container in 1 min.

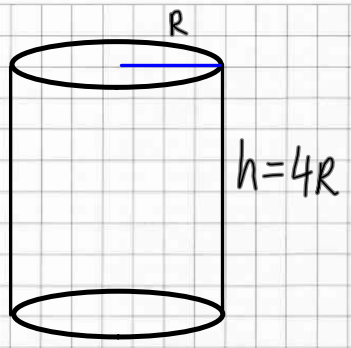
Together: they can fill $\frac{1}{3} + \frac{1}{15} = \frac{2}{5}$ in 1 min

or $\frac{1}{5}$ in 30 seconds

or $5/5$ in $(30\text{ sec})(5) = 2\frac{1}{2}$ min.

Answer: $2\frac{1}{2}$ minutes

(b) The height of a cylinder is four times its radius. If the volume of the cylinder is 108π cm^3, calculate the radius and height of the cylinder.



$V = \pi R^2 h$

$108\pi = \pi (R^2)(4R)$

$\frac{108}{4} = R^3$

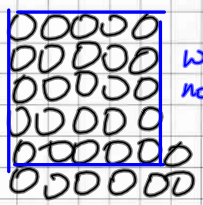
$R^3 = 27$

$R = \sqrt[3]{27} = 3$ cm

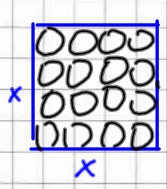
$h = 4(3) = 12$ cm

(c) 32 identical cylinders are packed into a square-based box. Calculate the surface area of the box.

Investigate possible layouts:

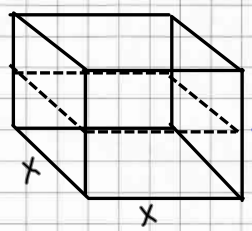


We see 1 layer is not viable



2 rows of 16 would work

$x = 8R = 8(3) = 24$ cm



$2h = 2(12) = 24$ cm

Box is cube $\Rightarrow$ SA $= 6(24) = 144$ cm^2

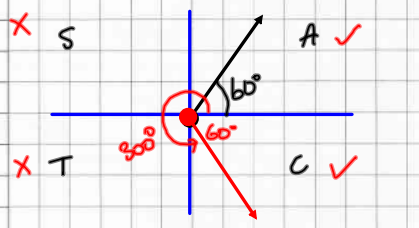
Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Solve $\cos\theta = 0.5$ for θ, where $0° \le \theta \le 360°$.

$\cos^{-1}(0.5) = 60°$



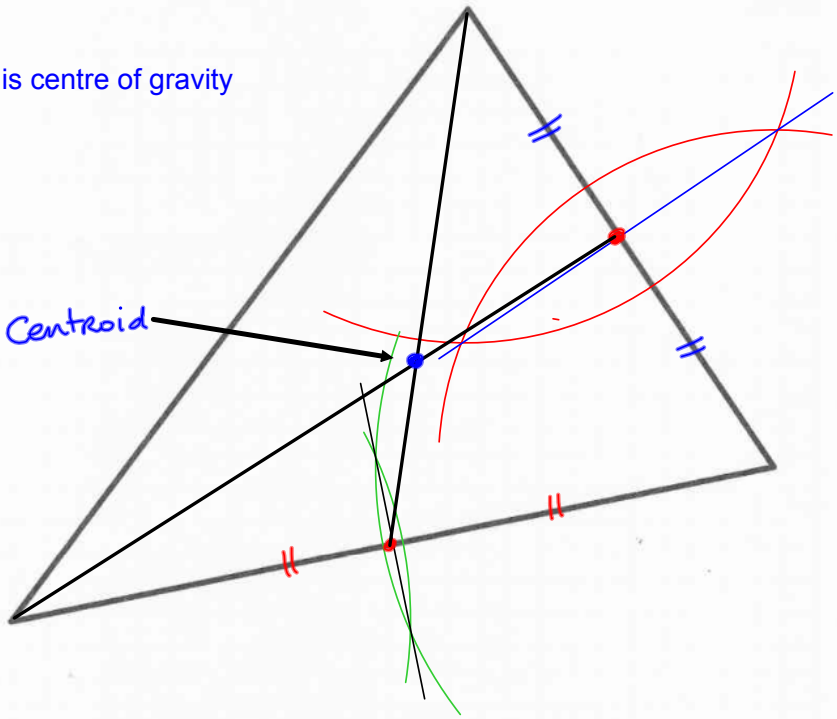
from diagram we see $\cos(300°) = 0.5$

Solution: $\theta = 60°$ or $300°$

$\cos\theta > 0$ in 4th quadrant

(b) A student is trying to find the centre of gravity of the following triangle. Use a suitable geometrical construction to find this point.

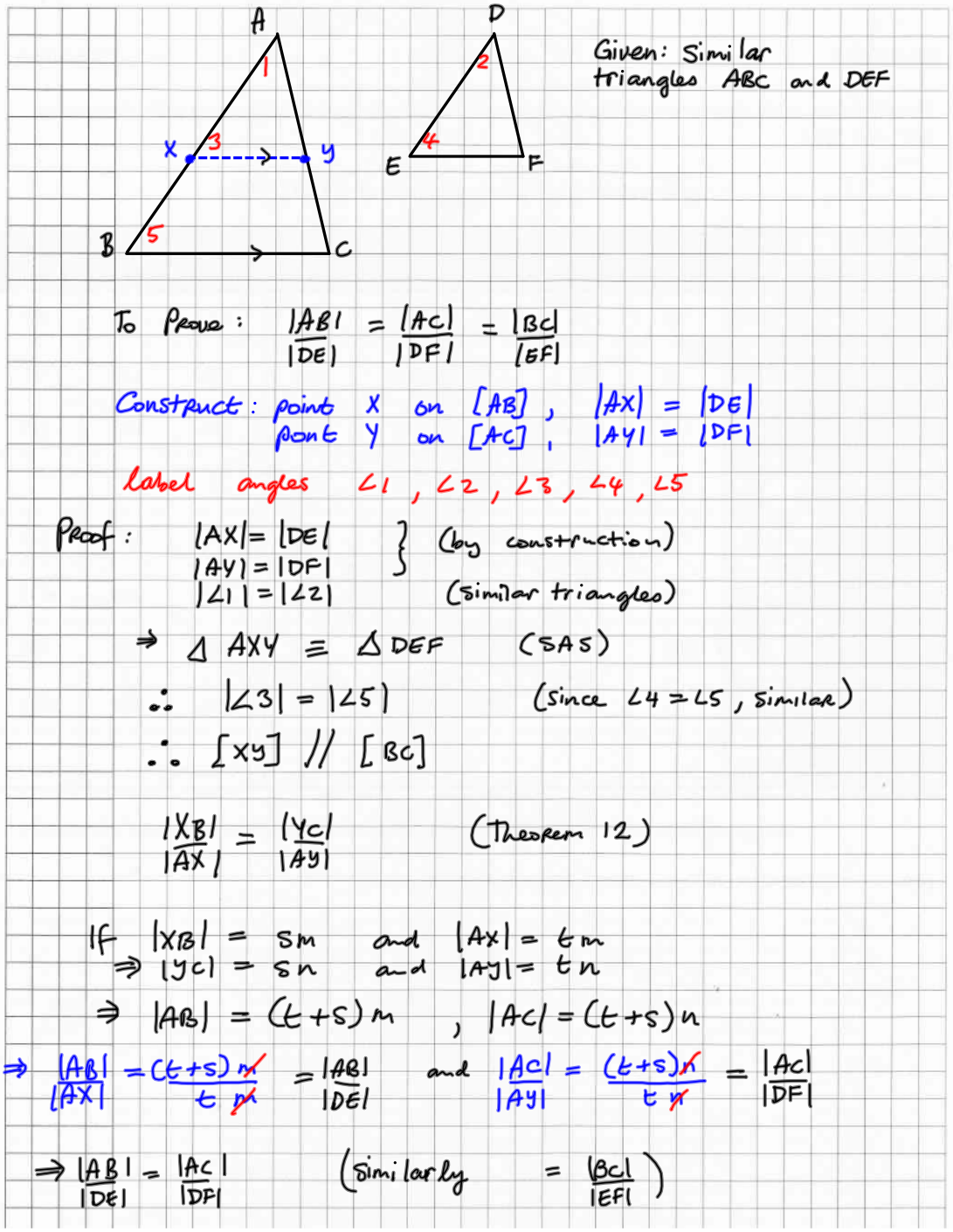
Centroid is centre of gravity



OR

Question 6B

Prove that if two triangles *ABC* and *DEF* are similar then their sides are in proportion.



Section B	**Contexts and Applications**	**150 marks**

Answer Question 7, Question 8 and Question 9.

Question 7 **(75 marks)**

The following table shows the numbers employed in certain sectors of industry in Ireland between 2005 and 2012.

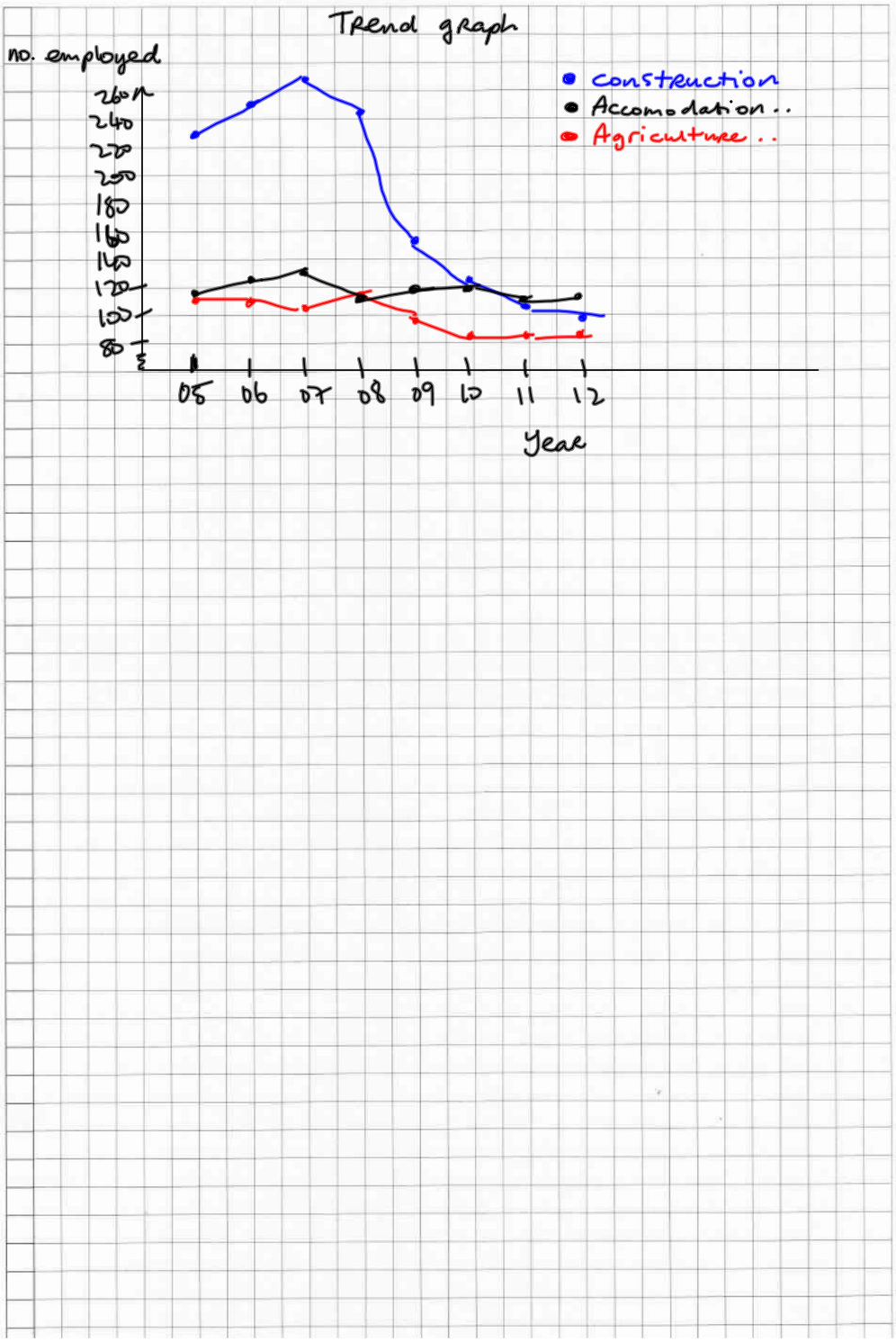
Employment and Unemployment (ILO) '000s

Economic Sector	Apr - Jun 05	Apr - Jun 06	Apr - Jun 07	Apr - Jun 08	Apr - Jun 09	Apr - Jun 10	Apr - Jun 11	Apr - Jun 12
Agriculture, Forestry & Fishing	110	110	109	115	97	85	86	87
Construction	228	252	270	241	155	125	106	99
Accommodation and food service activities	117	125	131	115	120	120	107	114
Information and communication	66	70	71	71	74	74	75	78
Education	126	136	141	146	150	150	147	144
Human health and social work activities	186	201	210	221	228	235	238	237
Total in Employment	**833**	**894**	**932**	**809**	**824**	**789**	**759**	**759**
Total Unemployed	96	98	103	127	265	294	305	309
Total Labour Force	929	992	1035	936	889	1083	1064	1068

(Adapted from: CSO http://www.cso.ie/en/statistics/labourmarket/principalstatistics)

(a) Calculate the total labour force for each Apr-Jun period.

(b) Choose a suitable graphical display to compare the employment figures in three of the sectors in the table above from 2005 to 2012.



(c) Discuss the shape and distribution of each sector you have chosen. In your opinion, why do you think these sectors have the distribution in the table?

Sector 1:

Construction : peaked in '07 and no.s employed in this sector dropped dramatically '08 to '12.

This is perhaps due to economy and less building happening.

Sector 2:

Agriculture ... : There was a slight reduction in no.s in this sector over the period. Perhaps parts of this sector are stable during short term recession eg.. farm owners continue to farm, forestory workers are semi-state employees etc..

Sector 3:

Accomodation ... : Has remained largely constant - perhaps this sector has retained employment in economic down turn by reducing wages / prices

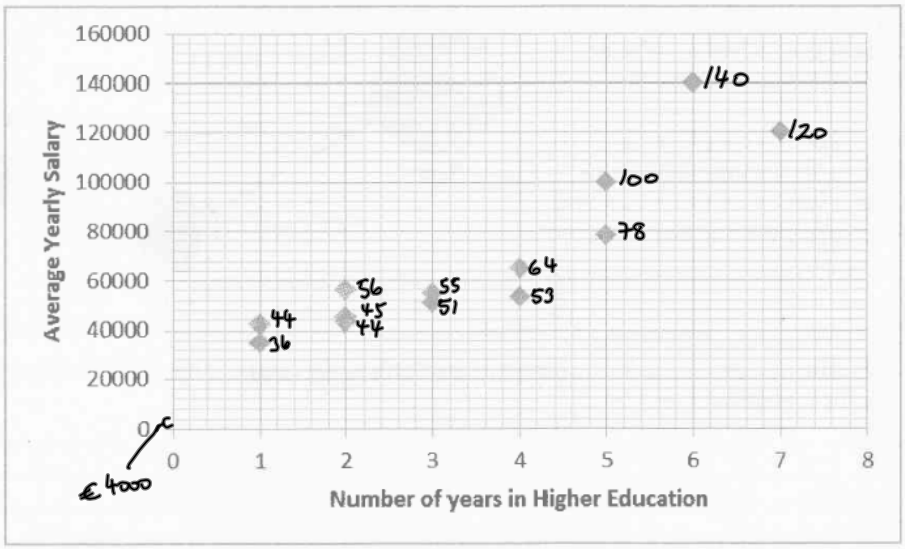
(d) Compare and contrast the figures in the construction and the education sector using a measure of central tendency and a measure of spread. Explain why you think the employment figures in both industries have followed the patterns in the table.

Construction : mean : 184.5
Standard deviation : 66.0
Range : 270 − 99 = 171

Education : mean : 142.5
Standard deviation : 7.6
Range : 150 − 126 = 24

Explanation:
After '07 less building has taken place while the no. of students has increased to record levels.

(e) Examine the following scatter plot.



(i) Describe the correlation coefficient of the plot.

Strong positive correlation

(ii) Calculate the correlation coefficient of the plot.

x	y	x	y
1	44	4	64
1	36	4	53
2	56	5	100
2	45	5	78
2	44	6	140
3	55	7	120
3	51		

$r = 0.89$

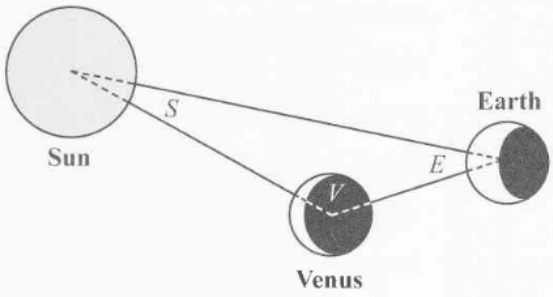
(iii) In your opinion, what is the plot attempting to show?

It shows the correlation between the no. of years spent in higher education and average annual salary.

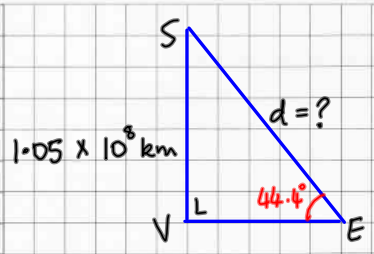
Question 8 **(50 marks)**

European astronomers carried out an experiment that started to measure the distance of Venus from the Sun.

When the centre of Venus is at right angles to both the centres of the Earth and the Sun it was found that Venus was 1.05×10^8 km from the Sun and that $|\angle SEV| = 44.4°$.



(a) Calculate the distance from the centre of the Earth to the centre of the Sun, correct to two significant figures.



$$\sin 44.4° = \frac{1.05\times10^8}{d}$$

$$d = \frac{1.05\times10^8}{\sin 44.4°}$$

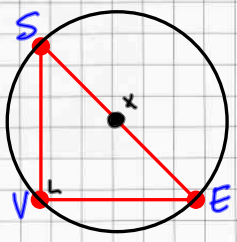
$$= 150,000,000 \text{ km}$$

$$= 1.5\times10^8 \text{ km}$$

A satellite is put in orbit so that it is equidistant from the centres of all three planets when they are positioned as above.

(b) Where would the satellite be located? Explain your answer fully.

Assume the sun is a planet

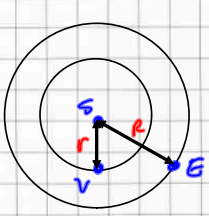


The circumcentre, X is equidistant to S, V. and E.

Since $|\angle SVE| = 90°$ $\Rightarrow$ X is midpoint of [SE]

(c) At what ratio does Venus orbit the Sun, compared with the Earth's orbit?

Assume circular orbits



$C = 2\pi r$

$C_V : C_E$

$2\pi r : 2\pi R$

$1.05 : 1.5$

$7 : 10$

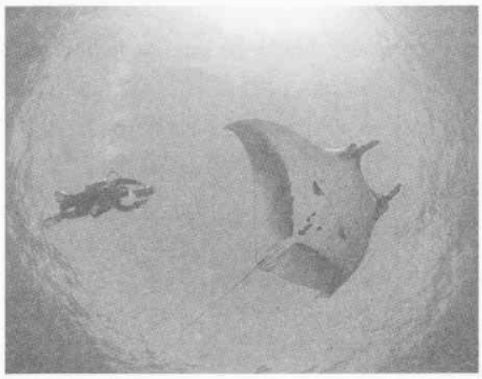
(d) Will this ratio hold for all positions of the three planets? Explain your answer fully.

yes • if orbits are circular

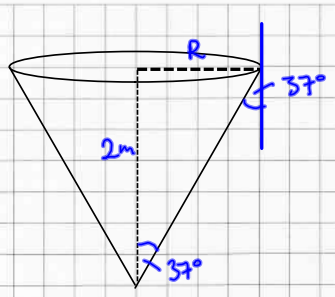
• Suns position is stationary relative to other planets.

Question 9 **(25 marks)**

(a) **Snell's window** is a phenomenon by which an underwater viewer sees everything above the surface through a cone of light.
It is caused by the refraction of light. Only light which strikes the surface of the water within a circle of a certain radius, r, will reach a point P below the surface.



A light at the bottom of a pool 2 m deep emits light upwards in all directions.
The maximum angle created by a ray of light and the vertical is 37°.
Calculate the radius of the disc through which the light leaves the surface of the water, correct to one decimal place.



$\tan 37° = R/2$

$R = 2 \tan 37°$

$\approx 1{\cdot}5$ m (1 dp)

(b) Calculate the volume of water contained in the cone, correct to one decimal place.

$V = \frac{1}{3}\pi R^2 h$

$\approx \frac{1}{3}(3.14)(1.5)^2(2)$

$\approx 4.7 \ m^3$ (1 d.p.)