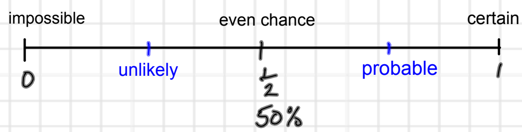
Probability Revision

ORDINARY LEVEL

Probability scale



The sum of probabilities of outcomes = 1

Probability of an event happening

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Sample Spaces

H = Head
T = Tail

Toss 2 coins

	H	T
H	HH	HT
T	TH	TT

Total when 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Relative Frequency of event in an experiment

Relative Frequency = Estimate of Probability

$$\text{Relative Frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

Mutually exclusive

Outcomes are mutually exclusive if they cannot happen at the same time

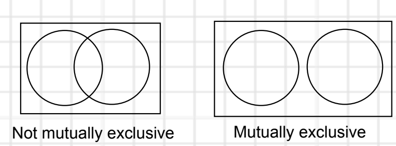
Addition Law

for mutually exclusive events

If event A and B are mutually exclusive then:

$$P\text{ (A or B)} = P(A) + P(B)$$

Use Venn diagrams



Independent events

Events are independent if the probability of one event is not affected by the other event happening or not happening.

Multiplication Rule
for independent events

the AND rule

If events A and B are independent then the probability of A and B both happening is:

$P(A \cap B) = P(A) \times P(B)$

Bernoulli Trials

S = SUCCESS

F = FAIL

$1 - P(S) = P(F)$

'Bernoulli' only applies when:

- outcomes can be classified as success or failure
- the probability of success is the same for each trial
- trails are independent of each other

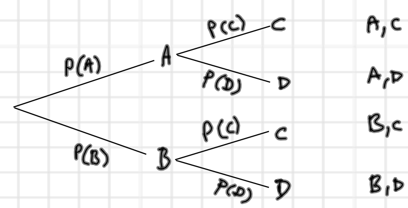
$$P(S,S,F) = P(S) \times P(S) \times P(F)$$
$$P(F,F,S) = P(F) \times P(F) \times P(S)$$

Tree diagrams

Event 1
with outcomes A or B

Event 2
with outcomes C or D

Possible Outcomes



Probabilities

$$P(A,C) = P(A) \times P(C)$$
$$P(A,D) = P(A) \times P(D)$$
$$P(B,C) = P(B) \times P(D)$$
$$P(B,D) = P(B) \times P(D)$$

Expected Value

The expected number of a particular outcome in an experiment equals the probability of the event times the number of trials.

$$E(x) = \sum x \cdot P(x)$$

Counting

The Fundamental Principle of Counting

If one task can be done in X ways and a second task in Y ways then the first task followed by the second can be done in XY ways.

e.g. Menu with choice of 3 starters, 4 mains and 2 desserts has how many possible meal choices?

Answer = $3 \times 4 \times 2 = 24$

Arrangements or Permutations

(order matters!)

$_nP_r$

The number of arrangements of n different objects is n!

e.g. how many 3 digit numbers can be made out of the numbers 1, 2 and 3?

ANSWER = $3! = 3 \times 2 \times 1 = 6$

e.g. how many 3 digit numbers can be made out of the numbers 1, 2, 3, 4, 5 and 6?

ANSWER = $\boxed{6} \times \boxed{5} \times \boxed{4} = 120$ $(= {}_6P_2)$

Selecting Combinations

$$_nC_r = \binom{n}{r}$$

"n choose r"

$$\binom{n}{r} = \binom{n}{n-r}$$

e.g. How many ways can a committee of 3 people be selected from a group of 10?

ANSWER = $\binom{10}{3} = 120$

e.g. How many combinations in a lottery in which you choose 4 numbers from 28?

ANSWER = $\binom{28}{4} = 20{,}475$

e.g. How many ways can 5 people be selected from a group of 6 women and 6 men if there must be at least 3 women on the committee?

$\Rightarrow$ options are $3W \cap 2M$
or $4W \cap 1M$
or $5W \cap 0M$

$$= \binom{6}{3} \times \binom{6}{2} + \binom{6}{4} \times \binom{6}{1} + \binom{6}{5} \times \binom{6}{0}$$

$$= 20 \times 15 + 15 \times 6 + 6 \times 1 = 396$$