Ouestion 9 (50 marks)

The atmospheric pressure is the pressure exerted by the air in the earth's atmosphere. It can be measured in kilopascals (kPa). The average atmospheric pressure varies with altitude: the higher up you go, the lower the pressure is.

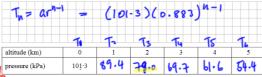
Some students are investigating this variation in pressure, using some data that they found on the internet. They have information about the average pressure at various altitudes.

Six of the entries in the data set are as shown in the table below:

	- 11	7.	73			
altitude (km)	0	1	2	3	4	5
pressure (kPa)	101-3	89-9	79.5	70.1	61.6	54.0

By looking at the pattern, the students are trying to find a suitable model to match the data

- (a) Hannah suggests that this is approximately a geometric sequence. She says she can match the data fairly well by taking the first term as 101.3 and the common ratio as 0.883.
 - (i) Complete the table below to show the values given by Hannah's model, correct to one





By considering the percentage errors in the above values, insert an appropriate number to complete the statement below.



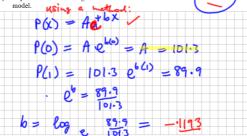
(b) Thomas suggests modelling the data with the following exponential function:

$$p = 101.3 \times e^{-0.1244h}$$

where p is the pressure in kilopascals, and h is the altitude in kilometres.

(i) Taking any one value other than 0 for the altitude, verify that the pressure given by Thomas's model and the pressure given by Hannah's model differ by less than 0:01 kPa

(ii) Explain how Thomas might have arrived at the value of the constant 0.1244 in I

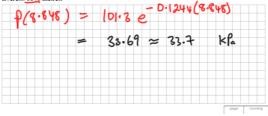


- (c) Hannah's model is *discrete*, while Thomas's is *continu*(i) Explain what this means.

Combinuous could be any number in a given range Where as Hannals Resufts are Correct to 1 deamer place

(ii) State one advantage of a continuous model over a discrete one





(e) Using Thomas's model, find an estimate for the altitude at which the atmospheric pressure is half of its value at sea level (altitude 0 km).

$$\frac{101.3}{2} = 50.65 \quad \text{Appension} \quad \text{kPa}$$

$$P(h) = 101.3 \quad \text{e}^{-1.244 \, h} = 50.65$$

$$e^{-0.1244 \, h} = \frac{50.65}{101.3} = \frac{1}{2}$$

$$-0.1244 \, h = \log_{10} \frac{1}{2} = -0.69$$

$$h = 0.69 / 0.1244 = 5.5 \quad \text{km}$$

let | floor = 3 hetres aprox = 0.003 km pressure at ground = 101.3 k/a fressure less | kra = 100.3 kfa P(h) = 101.3 e^{-D.1244} h = 100.3 $\Rightarrow -0.1244h = log_e(\frac{100.3}{101.3}) = -0.0099$ h = 0.0099/0.1244 = 0.0797 km ho floors = 0.0797/0.003 \$2.27 floors