**Question 9** **(50 marks)**

The *atmospheric pressure* is the pressure exerted by the air in the earth's atmosphere. It can be measured in kilopascals (kPa). The average atmospheric pressure varies with altitude: the higher up you go, the lower the pressure is.

Some students are investigating this variation in pressure, using some data that they found on the internet. They have information about the average pressure at various altitudes.

Six of the entries in the data set are as shown in the table below:

| | $T_1$ | $T_2$ | $T_3$ | | | |
|---|---|---|---|---|---|---|
| altitude (km) | 0 | 1 | 2 | 3 | 4 | 5 |
| pressure (kPa) | 101·3 | 89·9 | 79·5 | 70·1 | 61·6 | 54·0 |

By looking at the pattern, the students are trying to find a suitable model to match the data.

**(a)** Hannah suggests that this is approximately a geometric sequence. She says she can match the data fairly well by taking the first term as 101·3 and the common ratio as 0·883.

**(i)** Complete the table below to show the values given by Hannah's model, correct to one decimal place.

$$T_n = ar^{n-1} = (101\cdot3)(0.883)^{n-1}$$

| | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ |
|---|---|---|---|---|---|---|
| altitude (km) | 0 | 1 | 2 | 3 | 4 | 5 |
| pressure (kPa) | 101·3 | 89·4 | 79·0 | 69·7 | 61·6 | 54·4 |

Error ×100%
right answer

**(ii)** By considering the percentage errors in the above values, insert an appropriate number to complete the statement below.

"Hannah's model is accurate to within 0·63 %."

$$\frac{79\cdot5-79}{79} \times 100\%$$

**(b)** Thomas suggests modelling the data with the following exponential function:

$$\boxed{p = 101{\cdot}3 \times e^{-0{\cdot}1244h}}$$

where $p$ is the pressure in kilopascals, and $h$ is the altitude in kilometres.

**(i)** Taking any **one** value other than 0 for the altitude, verify that the pressure given by Thomas's model and the pressure given by Hannah's model differ by less than 0·01 kPa.

$$P(1) = 101\cdot3\, e^{-0\cdot1244(1)} = 89\cdot45 \approx 89\cdot5$$

Compared with Hannah's 89·4

**(ii)** Explain how Thomas might have arrived at the value of the constant 0·1244 in his model.

using a method:

$$P(x) = Ae^{+bx}$$

$$P(0) = Ae^{b(0)} = A = 101\cdot3$$

$$P(1) = 101\cdot3\, e^{b(1)} = 89\cdot9$$

$$e^b = \frac{89\cdot9}{101\cdot3}$$

$$b = \log_e \frac{89\cdot9}{101\cdot3} = -\cdot1193$$

**(c)** Hannah's model is *discrete*, while Thomas's is *continuous*.

**(i)** Explain what this means.

Continuous could be any number in a given range where as Hannah's results are correct to 1 decimal place

**(ii)** State one advantage of a continuous model over a discrete one.

more accurate

**(d)** Use Thomas's model to estimate the atmospheric pressure at the altitude of the top of Mount Everest: 8848 metres.

$$p(8\cdot848) = 101\cdot3\, e^{-0\cdot1244(8\cdot848)}$$

$$= 33\cdot69 \approx 33\cdot7 \text{ kPa}$$

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**(e)** Using Thomas's model, find an estimate for the altitude at which the atmospheric pressure is half of its value at sea level (altitude 0 km).

$\frac{101.3}{2} = 50.65$ Pressure kPa

$P(h) = 101.3\, e^{-.1244 h} = 50.65$

$e^{-0.1244 h} = \frac{50.65}{101.3} = \frac{1}{2}$

$-0.1244h = \log_e \frac{1}{2} = -0.69$

$h = 0.69/0.1244 = 5.5$ km

**(f)** People sometimes experience a sensation in their ears when the pressure changes. This can happen when travelling in a fast lift in a tall building. Experiments indicate that many people feel such a sensation if the pressure changes rapidly by 1 kilopascal or more. Suppose that such a person steps into a lift that is close to sea level. Taking a suitable approximation for the distance between two floors, estimate the number of floors that the person would need to travel in order to feel this sensation.

let 1 floor = 3 metres aprox = 0.003 km

pressure at ground = 101.3 kPa

Pressure less 1 kPa = 100.3 kPa

$P(h) = 101.3\, e^{-0.1244 h} = 100.3$

$\Rightarrow -0.1244h = \log_e \left(\frac{100.3}{101.3}\right) = -0.0099$

$h = 0.0099/0.1244 = 0.0797$ km

no. floors $= 0.0797/0.003 \approx 27$ floors