(c) *Acme Confectionery* has an employee pension plan. For an employee who qualifies for the full pension, *Acme Confectionery* will pay a sum of €20 000 on the day of retirement. It will then pay a sum on the same date each subsequent year for the next 25 years. Each year the employee is paid a sum that is 1% more than the amount paid in the previous year.
What sum of money must the company have set aside on the day of retirement in order to fund this pension? Assume an annual interest rate (AER) of 2·4%.

(c)	$20\,000+\frac{20\,000(1{\cdot}01)}{1{\cdot}024}+\frac{20\,000(1{\cdot}01^2)}{1{\cdot}024^2}+\frac{20\,000(1{\cdot}01^3)}{1{\cdot}024^3}+\cdots+\frac{20\,000(1{\cdot}01^{25})}{1{\cdot}024^{25}}$ $20\,000\left[1+\frac{1{\cdot}01}{1{\cdot}024}+\frac{1{\cdot}01^2}{1{\cdot}024^2}+\frac{1{\cdot}01^3}{1{\cdot}024^3}+\cdots+\frac{1{\cdot}01^{25}}{1{\cdot}024^{25}}\right]$ $a=1,\ r=\frac{1{\cdot}01}{1{\cdot}024}=\frac{505}{512},\ n=26$ $20000\left[\frac{\left(1-\frac{505^{26}}{512^{26}}\right)}{1-\frac{505}{512}}\right]=€440\,132{\cdot}40$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $20000(1{\cdot}01)$ or $\frac{20000}{1{\cdot}024}$ *Mid Partial Credit:* $\frac{20\,000(1{\cdot}01)}{1{\cdot}024}$ or similar term Correctly handles inflation element or completes correctly present values element and finishes *High Partial Credit:* GP with *a*, *r* and *n* identified **Note**: Treat $n=25$ as a misreading

Question 8 **(55 marks)**

(a) When a loan of €P is repaid in equal repayments of amount €A, at the end of each of t equal periods of time, where i is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$A = P\frac{i(1+i)^t}{((1+i)^t - 1)}$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

(b) Alex has a credit card debt of €5000. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is 2·5% of the original debt.

(i) What is the fixed monthly repayment, $€A$, required to pay the debt of €5000?

(ii) The annual percentage rate (APR) charged on debt by the credit card company is 21·75%, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of 21·75%.

(iii) Assume Alex pays the fixed monthly repayment, $€A$, each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5000 is reducing each month for the first three months, assuming an APR of 21·75%, charged and compounded monthly.

Payment number	Fixed monthly payment, $€A$	$€A$		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1			42·50	4957·50
2				
3				

(iv) Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of €5000, using the repayment method outlined at the beginning of **part (b)** above.
Give your answer in months, correct to the nearest month.

(v) Alex decides to borrow €5000 from the local Credit Union to pay off this credit card debt of €5000. The APR charge for the Credit Union loan is 8·5% fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.

(vi) How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from **part (b)(i)** each month to the credit card company?

<table>
<tr><td>(a)</td><td>

$$P = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \cdots\cdots + \frac{A}{(1+i)^t}$$

$$P = \frac{\left(\frac{A}{1+i}\right)\left(1-\left(\frac{1}{1+i}\right)^t\right)}{1-\frac{1}{1+i}}$$

$$= \frac{A\left(1-\frac{1}{(1+i)^t}\right)}{1+i-1}$$

$$= \frac{A((1+i)^t-1)}{i(1+i)^t}$$

$$A = \frac{P(i)(1+i)^t}{(1+i)^t-1}$$

</td><td>

Scale 5C (0, 3, 4, 5)

Low Partial Credit:

- $P = \frac{A}{1+i}$
- $A = P(1+i)$
- S_n formula with some substitution

High Partial Credit:

- full substitution for P (or A) into S_n formula.

</td></tr>
<tr><td>(b) (i)</td><td>

$2{\cdot}5\% \times 5000 = 125$

</td><td>

Scale 10B (0, 4, 10)

Partial Credit

- Any one unknown

</td></tr>
<tr><td>(b) (ii)</td><td>

$(1+i)^{\frac{1}{12}} = (1{\cdot}2175)^{\frac{1}{12}} = 1{\cdot}016535$

$Rate = 1{\cdot}65\%$

</td><td>

Scale 10B (0, 4, 10)

Partial Credit

- Formula with some substitution

</td></tr>
<tr><td>(b) (iii)</td><td colspan="2">

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82·50	42·50	4957·50
2	125	81·80	43·20	4914·30
3	125	81·09	43·91	4870·39

</td></tr>
<tr><td>(b) (iii)</td><td></td><td>

Scale 10C (0, 5, 8, 10)

Low Partial Credit:

- One correct additional entry

High Partial Credit:

- 6 correct additional entries

Note: Where interest rate in b(ii) is not 1·65%, then check the validity of all values given.

</td></tr>
</table>

(iv)	$$A = p\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$ $$A[(1+i)^t - 1] = pi(1+i)^t$$ $$A(1+i)^t - A = pi(1+i)^t$$ $$A = (1+i)^t[A - pi]$$ $$\frac{A}{A - pi} = (1+i)^t$$ $$\frac{125}{125 - 5000\left(\frac{1{\cdot}65}{100}\right)} = \left(1 + \frac{1{\cdot}65}{100}\right)^t$$ $$\frac{125}{42{\cdot}5} = (1{\cdot}0165)^t$$ $$\log\left(\frac{125}{42{\cdot}5}\right) = t\log(1{\cdot}0165)$$ $$t = \frac{\log\left(\frac{125}{42{\cdot}5}\right)}{\log(1{\cdot}0165)}$$ $$t = 65{\cdot}920$$ $$t = 66 \text{ months}$$ **OR** $$A = p\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$ $$125 = \frac{5000(0{\cdot}0165)(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$ $$125 = \frac{82{\cdot}5(1{\cdot}0165)^t}{(1{\cdot}0165)^t - 1}$$ $$\frac{125}{82{\cdot}5} = \frac{1{\cdot}0165^t}{1{\cdot}0165^t - 1}$$ $$\frac{50}{33} = \frac{1{\cdot}0165^t}{1{\cdot}0165^t - 1}$$ $$50(1 \cdot 0165^t - 1) = 33(1{\cdot}0165^t)$$ $$50(1{\cdot}0165^t) - 50 = 33(1{\cdot}0165^t)$$ $$50(1{\cdot}0165^t) - 33(1{\cdot}0165^t) = 50$$ $$1{\cdot}0165^t(50 - 33) = 50$$ $$1{\cdot}0165^t(17) = 50$$ $$1{\cdot}0165^t = \frac{50}{17}$$ $$t\log 1{\cdot}0165 = \log\frac{50}{17}$$ $$t = \frac{\log\left(\frac{50}{17}\right)}{\log 1{\cdot}0165} = 65{\cdot}92$$ $$t = 66 \text{ months}$$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Formula with some substitution • Some relevant manipulation of formula. *High Partial Credit:* • Equation in t (t no longer an index)

(v)	$$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$ $$= \frac{5000\left(1{\cdot}085^{\frac{1}{52}} - 1\right)(1{\cdot}085)^3}{(1{\cdot}085)^3 - 1}$$ $$= €36{\cdot}16$$ **OR** Weekly interest rate $(1+i)^{52} = 1{\cdot}085$ $$1 + i = 1{\cdot}085^{\frac{1}{52}}$$ $$1 + i = 1{\cdot}00157$$ $$i = 0{\cdot}00157$$ $$A = \frac{pi(1+i)^t}{(1+i)^t - 1}$$ $$A = \frac{5000(0{\cdot}00157)(1{\cdot}00157)^{156}}{(1{\cdot}00157)^{156} - 1}$$ $$= €36{\cdot}16$$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • r (weekly) found *High Partial Credit:* • Fully substituted equation
(vi)	$$125 \times 66 - (36{\cdot}16)(156)$$ =€2609·04	**Scale 5B (0, 3, 5)** *Partial Credit:* • Total repayment by either method found

A local sports club is planning to run a weekly lotto. To win the Jackpot of €1000, contestants must match one letter chosen from the 26 letters in the alphabet and two numbers chosen, in the correct order, from the numbers 0 to 9. In this lotto, repetition of numbers is allowed (e.g. M, 3, 3 is an outcome).

(a) Calculate the probability that M, 3, 3 would be the winning outcome in a particular week.

(b) If a contestant matches the letter only, or the letter and one number (but not both numbers), they will win €50. Using the table below, or otherwise, find how much the club should expect to make or lose on each play, correct to the nearest cent, if they charge €2 per play.

Event	**Payout** (x) €	**Probability** (P(x))	x.P(x)
Win Jackpot			
Match letter and first number only			
Match letter and second number only			
Match letter and neither number			
Fail to win			

(c) The club estimates that the average number of plays per week will be 845. If the club wants to make an average profit of €600 per week from the lotto, how much should the club charge per play, correct to the nearest cent?

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Q6	Model Solution – 25 Marks	Marking Notes
(a)	$P(M,3,3)=\frac{1}{26}\times\frac{1}{10}\times\frac{1}{10}=\frac{1}{2600}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • any correct relevant probability *High Partial credit* • correct probabilities but not expressed as single fraction or equivalent **Note:** Accept correct answer without supporting work
(b)	See tables below	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • 1 correct entry to table *High Partial Credit* • all entries correct but fails to finish or finishes incorrectly • no conclusion

Event	Payout	Prob (P(*x*))	*x*.P(*x*)
Win	1000	$\frac{1}{2600}$	$\frac{1000}{2600}$
letter 1 No.	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter 2nd No	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter only	50	$\frac{81}{2600}$	$\frac{4050}{2600}$
Fail to win	0		0

$$\sum x.P(x)=\frac{5950}{2600}=2{\cdot}29$$

Club loses 29 cent per play

Or

Event	Pay out	Prob (P(*x*)	*x*.P(*x*)
Win	-998	$^{1}/_{2600}$	$^{-998}/_{2600}$
letter + 1st No.	-48	$^{9}/_{2600}$	$^{-432}/_{2600}$
Letter + 2nd No	-48	$^{9}/_{2600}$	$^{-432}/_{2600}$
letter only	-48	$^{81}/_{2600}$	$^{-3888}/_{2600}$
Fail to Win	$+2$	$^{2500}/_{2600}$	$^{5000}/_{2600}$

$\sum x.P(x)=-\frac{750}{2600}=-29$ cent

(c)		
	Profit = Revenue – Pay-out $600 = 845(x - 2{\cdot}29)$ $$x = \frac{600 + 845(2{\cdot}29)}{845}$$ $$x = 3$$ **or** $$\frac{600}{845} = 0{\cdot}71$$ $$0{\cdot}71 + 2{\cdot}29 = 3$$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • links profit, revenue and payout *High partial Credit* • formula fully substituted

Data on earnings were published for a particular country. The data showed that the annual income of people in full-time employment was normally distributed with a mean of €39 400 and a standard deviation of €12 920.

(a) **(i)** The government intends to impose a new tax on incomes over €60 000.
Find the percentage of full-time workers who will be liable for this tax, correct to one decimal place.

(ii) The government will also provide a subsidy to the lowest 10 % of income earners.
Find the level of income at which the government will stop paying the subsidy, correct to the nearest euro.

(iii) Some time later a research institute surveyed a sample of 1000 full-time workers, randomly selected, and found that the mean annual income of the sample was €38 280.
Test the hypothesis, at the 5 % level of significance, that the mean annual income of full-time workers has changed since the national data were published.
State the null hypothesis and the alternative hypothesis.
Give your conclusion in the context of the question.

(b) The research institute surveyed 400 full-time farmers, randomly selected from all the full-time farmers in the country, and found that the mean income for the sample was €26 974 and the standard deviation was €5120.
Assuming that annual farm income is normally distributed in this country, create a 95 % confidence interval for the mean income of full-time farmers.

(c) It is known that data on farm size are not normally distributed.
The research institute could take many large random samples of farm size and create a sampling distribution of the means of all these samples.
Give one reason why they might do this.

(d) The research institute also carried out a survey into the use of agricultural land.
n farmers were surveyed.
If the margin of error of the survey was 4·5 %, find the value of n.

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\mu = 39400, \ \sigma = 12920$ $z = \frac{x-\mu}{\sigma} = \frac{60000 - 39400}{12920}$ $z = 1{\cdot}59$ $P(z > 1{\cdot}59) = 1 - P(z < 1{\cdot}59)$ $= 1 - 0{\cdot}9441 = 0{\cdot}0559$ $= 5{\cdot}59\%$ = 5·6%	Scale 10D (0, 3, 5, 8, 10) *Low Partial Credit* • μ and σ identified *Mid Partial Credit* • z = 1·59 *High Partial Credit* • identifies 0·9441
(a) (ii)	$P(z \le z_1) = 0{\cdot}9$ $z_1 = 1{\cdot}28$ $\Rightarrow z_2 = -1{\cdot}28$ $\Rightarrow \frac{x - 39400}{12920} = -1{\cdot}28$ $x = 22862{\cdot}40$ = €22 862	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • identifies 1·28 but fails to progress *High Partial Credit* • formula for x fully substituted
(a) (iii)	$\mu = 39400, \ \sigma = 12920,$ $\bar{x} = 38280, \ n = 1000$ $H_0 \Rightarrow \mu = 39400$ $H_1 \Rightarrow \mu \neq 39400$ $z = \frac{38280 - 39400}{\frac{12920}{\sqrt{1000}}} = -2{\cdot}74$ $-2{\cdot}74 < -1{\cdot}96$ Result is significant. There is evidence to reject the null hypothesis The mean income has changed.	Scale 15D (0, 4, 7, 11,15) *Low Partial Credit* • z formulated with some substitution • states null and/or alternative hypothesis only • reference to 1·96 Mid Partial Credit • z fully substituted *High Partial Credit* • $z = -2{\cdot}74$ and stops • fails to state the null and alternative hypothesis correctly • fails to contextualise the answer

or

Confidence Interval:

$$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$

$$39400 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$

$$[38599{\cdot}2,\ 40200{\cdot}8]$$

38280 outside range

Result is significant. There is evidence to reject the null hypothesis

The mean income has changed.

or

Confidence Interval:

$$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$

$$38280 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$

$$38280 \pm 1{\cdot}96(408{\cdot}57)$$

$$[37479{\cdot}2,\ 39080{\cdot}8]$$

39400 outside range

Result is significant. There is evidence to reject the null hypothesis

The mean income has changed.

(b)	$26974 - 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right) \le \mu$ $\le 26974 + 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right)$ $26472{\cdot}24 \le \mu \le 27475{\cdot}76$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • interval formulated with some correct substitution *High Partial Credit* • interval formulated with fully correct substitution
(c)	The distribution of sample means will be normally distributed	Scale 5B (0, 2, 5) *Partial Credit* • mentions 30 (or more) but not contextualised
(d)	$\frac{1}{\sqrt{n}} = 0{\cdot}045$ $\frac{1}{0{\cdot}045} = \sqrt{n}$ $n = \left(\frac{1}{0{\cdot}045}\right)^2 = 493{\cdot}827$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • $\frac{1}{\sqrt{n}}$ *High Partial Credit* • n formulated with fully correct substitution **Note:** Accept 493 farmers or 494 farmers

Question 6 **(25 marks)**

(a) Donagh is arranging a loan and is examining two different repayment options.

(i) Bank A will charge him a monthly interest rate of 0·35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·35%.

(ii) Bank B will charge him a rate that is equivalent to an APR of 4·5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4·5%.

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

(a) Donagh is arranging a loan and is examining two different repayment options.

(i) Bank A will charge him a monthly interest rate of 0·35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·35%.

$$F = P(1+i)^t = 1(1+0\cdot 0035)^{12} = 1\cdot 042818$$
$$\Rightarrow i = 4\cdot 28\,\%$$

(ii) Bank B will charge him a rate that is equivalent to an APR of 4·5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4·5%.

$$F = P(1+i)^t$$
$$1\cdot 045 = 1(1+i)^{12}$$
$$1+i = \sqrt[12]{1\cdot 045} = 1\cdot 0036748$$
$$\Rightarrow i = 0\cdot 367\,\%$$

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

$$A = P\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$

$$= 80000\left[\frac{0\cdot0035(1\cdot0035)^{120}}{(1\cdot0035)^{120} - 1}\right]$$

$$= 80000\left[\frac{0\cdot00532296}{0\cdot520846}\right]$$

$$= 817\cdot59 = €818$$

or

$$80000 = \frac{A}{1\cdot0035} + \frac{A}{1\cdot0035^2} + \ldots + \frac{A}{1\cdot0035^{120}}$$

$$= A\left[\frac{1}{1\cdot0035} + \frac{1}{1\cdot0035^2} + \ldots + \frac{1}{1\cdot0035^{120}}\right]$$

$$= A\left[\frac{\frac{1}{1\cdot0035}\left(1-\left(\frac{1}{1\cdot0035}\right)^{120}\right)}{1-\frac{1}{1\cdot0035}}\right]$$

$$= A\left[\frac{0\cdot342471198}{0.0035}\right]$$

$$= A\left[97\cdot8489137\right]$$

$$A = 817\cdot58 = €818$$

Question 8 **(50 marks)**

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(a) Write down the present value of a future payment of €20,000 in one year's time.

(b) Write down, in terms of t, the present value of a future payment of €20,000 in t years' time.

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the fund required.

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.

(ii) Write down, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

(iii) If Pádraig makes 480 equal monthly payments of €P from now until his retirement, what value of P will give the fund he requires?

(e) If Pádraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?

page	running

We have

$$P = \frac{F}{1+i} = \frac{20000}{1.03} = 19417.48$$

So the present value is €19417.48 to the nearest cent.

(b) Write down, in terms of t, the present value of a future payment of €$20,000$ in t years' time.

We have

$$P = \frac{F}{(1+i)^t} = \frac{20000}{(1.03)^t}.$$

(c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €$20,000$ at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the final fund required.

Using the solution to part (b), we see that the amount required on the date of requirement is given by

$$A = 20000 + \frac{20000}{1.03} + \cdots \frac{20000}{(1.03)^{24}}$$

Using the notation of the formula on page 22 of the Formula and Tables book, we have a geometric series with $a = 20000$, $r = \frac{1}{1.03}$ and $n = 25$. Therefore

$$A = \frac{20000\left(1 - \left(\frac{1}{1.03}\right)^{25}\right)}{1 - \frac{1}{1.03}}$$

Using a calculator we obtain

$$A = €358711$$

to the nearest euro.

(d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part **(c)**. His retirement is $40 \times 12 = 480$ months away.

(i) Find, correct to four significant places, the rate of interest per month that would, if paid and compounded annually, be equivalent to an effective annual rate of 3%.

We must solve $(1+i)^{12} = 1.03$. So $(1+i) = \sqrt[12]{1.03}$. Therefore

$$i = \sqrt[12]{1.03} - 1 = 0.002466$$

correct to 4 significant places. So the answer is

$$0.2466\%.$$

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(ii) Write dowm, in terms of n and P, the value on the retirement date of a payment of €P made n months before the retirement date.

Using the formula on page 30 of the Formula and Tables booklet we obtain

$$P(1.002466)^n.$$

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(iii) If Pádraig makes 480 equal payments of €P from now until his retirement, what value of P will give him the fund he requires?

We must solve

$$P(1.002466)^{480} + P(1.002466)^{479} + \cdots + P(1.002466) = 358711$$

or

$$P(1.002466 + (1.002466)^2 + \cdots + (1.002466)^{480}) = 358711$$

Using the formula for the sum of a geometric series, we obtain

$$P\left(\frac{1.002466(1-(1.002466)^{480})}{1-1.002466}\right) = 358711$$

or

$$P(919.38) = 358711.$$

Therefore $P = \frac{358711}{919.38} =$ €390.17 to the nearest cent.

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(e) If Pádraig waits for ten years before starting his pension fund, how much will he then have to pay each month in order to generate the same pension fund?

Now the number of months until his retirement date is $30 \times 12 = 360$. So as above we must solve

$$P(1.002466)^{360} + P(1.002466)^{359} + \cdots + P(1.002466) = 358711$$

or

$$P(1.002466 + (1.002466)^2 + \cdots + (1.002466)^{360}) = 358711$$

Using the formula for the sum of a geometric series, we obtain

$$P\left(\frac{1.002466(1-(1.002466)^{360})}{1-1.002466}\right) = 358711$$

or

$$P(580.11) = 358711.$$

Therefore, in this case, $P = \frac{358711}{580.11} =$ €618.35 to the nearest cent.

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(a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.

(i) Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0·327%, correct to 3 decimal places.

(ii) Niamh has €15 000 in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?

$$(1+i)^{12} = 1\cdot 04 \Rightarrow 1+i = \sqrt[12]{1\cdot 04} = 1\cdot 003273 \Rightarrow i = 0\cdot 003274$$

Hence, $i = 0{\cdot}327\%$

OR

$$(1.00327)^{12} = 1{\cdot}039953481$$
$$= 1{\cdot}0400$$

$r = 4\%$

$$15000 = P\left(1\cdot 00327^{36} + 1\cdot 00327^{35} + \cdots + 1\cdot 00327^{2} + 1\cdot 00327\right)$$

$$\Rightarrow P\left[\frac{1\cdot 00327\left(1\cdot 00327^{36} - 1\right)}{1\cdot 00327 - 1}\right] = 15000$$

$$\Rightarrow P[38\cdot 26326387] = 15000$$

$$\Rightarrow P = 392\cdot 02 \;=\; €392$$

(b) Conall borrowed to buy a car. He borrowed €15 000 at a monthly interest rate of 0·866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payments?

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

$$= 15000\left[\frac{0\cdot 00866(1+0\cdot 00866)^{36}}{1\cdot 00866^{36} - 1}\right]$$

$$= 486\cdot 77$$

Monthly payment €487

OR

$$15000 = P\left(\frac{1}{1\cdot 00866} + \frac{1}{1\cdot 00866^2} + \cdots + \frac{1}{1\cdot 00866^{36}}\right)$$

$$\Rightarrow P\left[\frac{\frac{1}{1\cdot 00866}\left(1 - \frac{1}{1\cdot 00866^{36}}\right)}{1 - \frac{1}{1\cdot 00866}}\right] = 15000$$

$$\Rightarrow P[30\cdot 8151777] = 15000$$

$$\Rightarrow P = 486\cdot 77$$

Monthly payment €487